

Strictly As Per
Boards
New Syllabus

STATE BOARD

महाराष्ट्रात
प्रथमच...

11th आणि 12th (Science) ची पुस्तके
English व Scientific मराठी मधून...

My Favorite Guide on

MATHEMATICS & STATISTICS

STD - 12th (Science)

With Solutions of
Board Question Papers

Feb / March - 2022
Feb / March - 2023
Feb / March - 2024
Feb / March - 2025

आता...
मिळवा
80 पैकी
80

फक्त
ज्ञानदा
मधून



ज्ञानविमान वाजवा... ज्ञानमय मराठीचा...

DNYANADA PUBLICATION, PUNE

Author : Prof. Prajakta Prashant Bhaskare
MSc. (Mathematics) - PAH Solapur University

Authors :
Qualified and Experienced Staffs From :
PUNE | LATUR PATTERN

TOPIC - 01

MATHEMATICAL LOGIC

Logical connectives Simple and Compound Statements:

i) Simple statements: A statement or proposition whose truth value does not depend on another statement is called a simple or atomic statement.

ii) Compound statement: If a statement is combination of two or more simple statements, then it is said to be a compound statement.

iii) Component statement: The simple statements are called component statement.

iv) Logical variables: Statements are represented by the letters as p, q, r, s.... These letters are called logical variables.

Ex. i) p = the sun sets in the East.

ii) q = Sum of the angles of a triangle is 180° .

v) Logical connectives: The connectives joining two or more simple statements are called logical connectives. There are five connectives in mathematical logic.

Sr. No.	Connectives	Term used in Compound statement	Symbol
1	Not	negation	$\sim$
2	And	conjunction	$\wedge$
3	Or	disjunction	$\vee$
4	If...then	implication/conditional	$\rightarrow$
5	If and only if	Double implication/Bi-conditional	$\leftrightarrow$

i) Simple statements: ज्या statement ची truth value इतर कोणत्याही statement च्या truth value वर depend (अवलंबून) राहत नाही त्याला simple statement म्हणतात.

ii) Compound statement- जर एखादे statement हे दोन किंवा अधिक simple statement पासून तयार होते, त्याला compound statement म्हणतात.

iii) Component statement (पुरक विधान)- Simple statement ला Component statement म्हणतात.

iv) Logic variables - statement represent करण्यासाठी जी letters वापरली जातात, त्यांना logical variables म्हणतात. खालील examples पहा.

v) Logical connectives: जी connective words, logical statement join करण्यासाठी वापरली जातात, त्यास logical connective म्हणतात. विविध connectives, त्यांची नावे आणि symbols खालील table मध्ये दिली आहेत, त्याचे अवलोकन करा.

Truth Table:

A table showing the truth values, of the compound statements is called a **truth table**.

It has a number of columns and rows.

Note: for n statements, there are $(2)^n$ rows.

Some connectives:

1) Conjunction: Any two statements can be connected by the word "and" to form a compound statement is called the conjunction of the original statements.

In other words if p, q are two statements then 'p and q' is called as conjunction. It is denoted by ' $p \wedge q$ ' and it is read as 'p conjunction q' or 'p and q'. The conjunction $p \wedge q$ is said to be true if and only if both p and q are true.

Truth table for conjunction: ($p \wedge q$)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2) Disjunction (or Alternation):

Any two statements can be connected by the word "or" to form a compound statement is called the disjunction of the original statements.

In other words if p, q are two statements then ' p or q ' is called as disjunction. It is denoted by ' $p \vee q$ ' and it is read as ' p or q ' or ' p disjunction q '.

The disjunction $p \vee q$ is false if and only if both p and q are false.

Truth table for Disjunction: ($p \vee q$)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3) Implication or conditional statement:

Any two statements can be connected by the word "Ifthen..." to form a compound statement is called implication.

In other words if p, q are two statements then ' p then q ' is called as conditional. It is denoted by $p \rightarrow q$ or $p \Rightarrow q$ and it is read as ' p implies q ' or ' p then q '.

The conditional statement $p \rightarrow q$ is False only if p is true and q is false. Otherwise it is true. Here p is called hypothesis or antecedent and q is called conclusion or consequence.

Truth table for Implication: ($p \rightarrow q$)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

4) Double implication or Bi-conditional:

Any two statement can be connected by the word "if and only if" to form a compound statement is called biconditional.

In other words if p, q are two statements then ' p if and only if q ' is called as biconditional. It is denoted by ' $p \leftrightarrow q$ ' or ' $p \Leftrightarrow q$ '. It is read as ' p biconditional q ' or ' p if and only if q '.

Biconditional statement $p \leftrightarrow q$ is true if p and q have same truth values. Otherwise it is False.

Truth table for Double implication: ($p \leftrightarrow q$)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

5) Negation: If p is a statement then the "not p " is called negation of statement p and it is denoted by " $\sim p$ ".

Note: Negation of negation of a statement is the statement itself. That is, $\sim(\sim p) = p$

Truth table for Negation: ($\sim p$)

p	$\sim p$
T	F
F	T

Truth table:

ज्या table मध्ये दिलेल्या compound statement च्या Truth value व्यक्त होतात, त्यास truth table म्हणतात.

1) Conjunction:

जर दिलेले दोन statement join करण्यासाठी and हा word वापरला असेल तर त्या logical connectives ला conjunction म्हणतात. Conjunction साठी ($\wedge$) हा symbol वापरतात. जर p and q हे दोन्ही T असेल तर $p \wedge q$ ची Truthvalue T असते otherwise F असते.

2) Disjunction (or Alternation):

जर दिलेली दोन statements connect करण्यासाठी or हे connective वापरले असेल तर, त्या statement ला disjunction म्हणतात. जर p and q या दोन्ही statement ची value जर F असेल तर $p \vee q$ ची Truth value F असते otherwise T असते.

3) Implication or conditional statement:

जर दिलेली दोन statement join करण्यासाठी If ----- then चा उपयोग केला असेल तर त्या compound statement ला implication किंवा conditional statement म्हणतात.

जर p या statement ची value T आणि q या statement ची value F असेल तर $p \rightarrow q$ ची truth value F असते otherwise T असते.

4) Double implication or Bi-conditional:

जर दिलेली दोन statements connect करण्यासाठी If and only if हे connective word वापरले असतील, तर त्या statement ला Double implication किंवा bi-conditional म्हणतात.

TOPIC - 04

PAIR OF STRAIGHT LINES

Introduction:

Important Results and formulae:

1) Distance formula:

i) The distance of the point $P(x, y)$ from the origin $O(0,0)$ is given by $l(OP) = \sqrt{x^2 + y^2}$

ii) The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$l(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2) Section formula: (विभाजनाचे सूत्र)

i) If the point $P(x, y)$ divides the line segment AB internally in the ratio $m : n$ where

$$A(x_1, y_1) \text{ and } B(x_2, y_2) \text{ then } x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n}$$

ii) If the point $P(x, y)$ divides the line segment AB externally in the ratio $m : n$ where

$$A(x_1, y_1) \text{ and } B(x_2, y_2) \text{ then } x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$$

iii) Mid-point formula: (मध्य बिंदुचे सूत्र)

If $P(x, y)$ is the midpoint, of segment AB then

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Where $A(x_1, y_1)$ and $B(x_2, y_2)$

iv) Centroid formula: (मध्यगा संपात बिंदुचे सूत्र)

If $G(x, y)$ is centroid of $\triangle ABC$, where $B(x_2, y_2)$ and $C(x_3, y_3)$ then

$$x = \frac{x_1 + x_2 + x_3}{3}, y = \frac{y_1 + y_2 + y_3}{3}$$

Distance formula (अंतराचे सूत्र):

i) केंद्रबिंदुपासून $p(x, y)$ चे अंतर काढण्याचे सूत्र. $l(OP) = \sqrt{x^2 + y^2}$ हे आहे.

ii) $A(x_1, y_1)$ and (x_2, y_2) या दोन point मधील अंतर काढण्याचे सूत्र

$$l(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ हे आहे.}$$

Section formula: (विभाजनाचे सूत्र):

i) जर point $P(x, y)$ हा line segment AB ला internally (आंतरभागात) $m:n$ या ratio मध्ये विभागत असेल तर, $A(x_1, y_1)$ and $B(x_2, y_2)$ then

$$x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + ny_1}{m+n} \text{ हे मिळते.}$$

ii) जर point $P(x, y)$ हा line segment AB ला externally (बाह्यभागात) $m:n$ या ratio मध्ये विभागत असेल तर, $A(x_1, y_1)$ and $B(x_2, y_2)$ then $x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$ हे मिळते.

Mid-point formula: (मध्य बिंदुचे सूत्र): जर point $P(x, y)$ हा line segment AB चा मध्यबिंदु असेल, तर

$$x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}$$

Centroid formula: (मध्यगा संपात बिंदुचे सूत्र):

जर point $G(x, y)$ हा triangle ABC चा Centroid (मध्यगा संपात) असेल तर,

$$x = \frac{x_1 + x_2 + x_3}{3}, y = \frac{y_1 + y_2 + y_3}{3}$$

3) i) Any point on x-axis is $(\pm a, 0)$.

ii) Any point on y-axis is $(0, \pm b)$.

4) i) The inclination of a line is the smallest positive angle made by the line, with the positive direction of the x-axis.

a) The inclination of the x-axis or a line parallel to the x-axis is 0° .

b) The inclination of the y-axis or a line parallel to the y-axis is 90° .

c) The inclination of the line bisecting the angles of the first and the third quadrant is 45° .

d) The inclination of the line bisecting the angles of the second and the fourth quadrants is 135° .

जर दिलेल्या line चे inclination (चढ / कल) हा line शी smallest positive angle, positive direction ने करत असेल तर

a) X-axis चे inclination किंवा x-axis ला parallel असणाऱ्या line चे inclination 0° असते.

b) Point a) प्रमाणेच y-axis चे inclination 0° असते.

c) First & third quadrant मधील कोनाचे (Angle) दुभाजक करणाऱ्या line चे inclination 45° असते.

d) Second & fourth quadrant मधुन angle ला bisect (दुभाजक) करुन जाणाऱ्या line चे inclination 135° असते.

Solved Examples

Ex.11) What is the distance from the point (2,3,4) to
i) the XY plane? ii) the X-axis?
iii) origin iv) point (-2, 7, 3)

Solution:

a) The distance from (2,3,4) to the XY plane is $|z| = 4$ units.

b) The distance from (2,3,4) to X-axis is $\sqrt{y^2 + z^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ units.

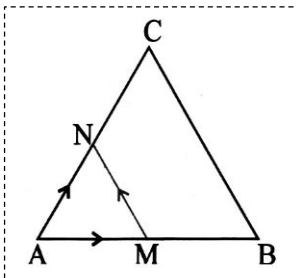
c) The distance from $(x,y,z) \equiv (2,3,4)$ to origin $(0,0,0)$ is $\sqrt{x^2 + y^2 + z^2} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$ units.

d) The distance from (2,3,4) to (-2,7,3) is

$$\sqrt{(2+2)^2 + (3-7)^2 + (4-3)^2} = \sqrt{16 + 16 + 1} = \sqrt{33} \text{ units.}$$

Ex.12) Prove that the line segment joining the midpoints of two sides of a triangle is parallel to and half of the third side.

Solution: Let the triangle be ABC. If M and N are the midpoints of AB and AC respectively, then $\vec{AM} = \frac{1}{2}\vec{AB}$ and $\vec{AN} = \frac{1}{2}\vec{AC}$. Thus by triangle law

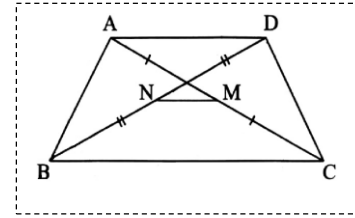


$$\vec{AN} = \vec{AM} + \vec{MN} \quad \therefore \vec{MN} = \vec{AN} - \vec{AM} = \frac{\vec{AC} - \vec{AB}}{2} = \frac{\vec{BC}}{2}$$

Thus, $\vec{MN}$ is parallel to and half as long as $\vec{BC}$.

Ex.13) In quadrilateral ABCD, M and N are the midpoints of the diagonals AC and BD respectively. Prove that $\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{MN}$

Solution:



$$\vec{AB} = \vec{AM} + \vec{MN} + \vec{NB} \quad \dots (1)$$

$$\vec{AD} = \vec{AM} + \vec{MN} + \vec{ND} \quad \dots (2)$$

$$\vec{CB} = \vec{CM} + \vec{MN} + \vec{NB} \quad \dots (3)$$

$$\vec{CD} = \vec{CM} + \vec{MN} + \vec{ND} \quad \dots (4)$$

Add (1),(2),(3),(4) to get

$$\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 2\vec{AM} + 2\vec{CM} + 4\vec{MN} + 2\vec{NB} + 2\vec{ND}$$

$$= 2(\vec{AM} + \vec{CM}) + 4\vec{MN} + 2(\vec{NB} + \vec{ND})$$

$$= 2(\vec{AM} - \vec{AM}) + 4\vec{MN} + 2(\vec{NB} - \vec{NB})$$

$$(\because \vec{MC} = \vec{AM} \text{ and } \vec{DN} = \vec{NB})$$

$$= 2 \times (\vec{0}) + 4\vec{MN} + 2(\vec{0}) = 4\vec{MN}$$

विद्यार्थी मित्रांनो वरील ex. मध्ये आपल्याला quadrilateral ABCD दिलेला आहे. त्यानुसार point M, N, हे diagonal AC व BD चे mid point असलेली Dia काढून घेणे तरी या सर्व माहितीवरून vector च्या basic properties चा use करून वरील example prove करावे.

Ex.14) Express $-\hat{i} - 3\hat{j} + 4\hat{k}$ as the linear combination of the vector $2\hat{i} + \hat{j} - 4\hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} - 2\hat{k}$.

Solution:

$$\text{Let } \vec{r} = -\hat{i} - 3\hat{j} + 4\hat{k} \quad \vec{a} = 2\hat{i} + \hat{j} - 4\hat{k}, \vec{b} = \hat{i} - \hat{j} + 3\hat{k} \quad \vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\text{Consider } \vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$$

$$-\hat{i} - 3\hat{j} + 4\hat{k} = x(2\hat{i} + \hat{j} - 4\hat{k}) + (2\hat{i} - \hat{j} + 3\hat{k}) + z(3\hat{i} + \hat{j} - 2\hat{k})$$

$$-\hat{i} - 3\hat{j} + 4\hat{k} = (2x + 2y + 3z)\hat{i} + (x - y + z)\hat{j} + (-4x + 3y - 2z)\hat{k}$$

$$\text{By equality of vectors, we get } -1 = 2x + 2y + 3z, -3 = x - y + z, 4 = -4x + 3y - 2z,$$

$$\text{Using Cramer's rule we get, } x = 2, y = 2, z = 3.$$

$$\text{Therefore } \vec{r} = 2\vec{a} + 2\vec{b} - 3\vec{c}$$

Solved Examples

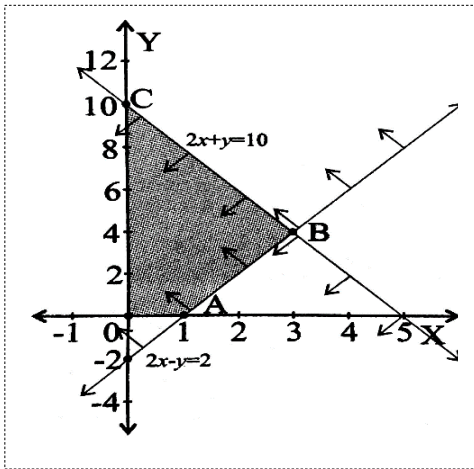
Ex. 1: Find the graphical solution of the system of inequations.

$$2x + y \leq 10, 2x - y \leq 2, x \geq 0, y \geq 0$$

Solutions: To find the solution of the system of given inequations -

Draw lines	x	y	Line passes through (x, y)	sign	Region
$L_1 2x + y = 10$	0	10	(0,10)	$\leq$	origin side L_1
	5	0	(5,0)		
$L_2 2x - y = 2$	0	-2	(0,-2)	$\leq$	Origin side L_2
	1	0	(1,0)		

The common shaded region OABCO is the graphical solution. This graphical solution is known as feasible solution.



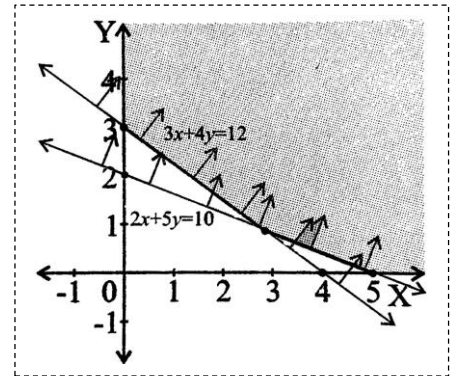
विद्यार्थी मित्रांनो वरील example मध्ये आपल्याला दोन line चे equation दिले आहेत त्यासाठी x आणि y intersect करून A व B असे दोन point काढून घेणे व त्यापासून line draw करणे असे दोन्ही equation साठी करणे व त्यानुसार origin test करून त्यांचा common region find करणे व तो shaded करणे. त्याला आपण graphical solution असे म्हणतो आणि याच graphical solution ला feasible solution असे म्हणतो

Remark: The restrictions $x \geq 0, y \geq 0$, are called non-negativity constraints.

Ex.2: Find the feasible solution of the system of inequations $3x + 4y \geq 12, 2x + 5y \geq 10, x \geq 0, y \geq 0$.

To draw (Inequations)	Draw line	x	y	Line passes through (x, y)	Sign	Region
$3x + 4y \geq 12$	$L_1 3x + 4y = 12$	0	3	(0,3)	$\geq$	Non-origin side of Line L_1
		4	0	(4,0)		
$2x + 5y \geq 10$	$L_2 2x + 5y = 10$	0	2	(0,2)	$\geq$	Non-origin side of Line L_2
		5	0	(5,0)		

Solution: Common shaded region is the feasible solution.



वरील example प्रमाणे solve करावे.

Miscellaneous Exercise - 4

I) Choose the correct option from the given alternatives:

1) $\int_2^3 \frac{dx}{x(x^3-1)} =$

A) $\frac{1}{3} \log\left(\frac{208}{189}\right)$ B) $\frac{1}{3} \log\left(\frac{189}{208}\right)$ C) $\log\left(\frac{208}{189}\right)$ D) $\log\left(\frac{189}{208}\right)$

2) $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x \cdot dx}{(1+\cos x)^2} =$

A) $\frac{4-\pi}{2}$ B) $\frac{\pi-4}{2}$ C) $4 - \frac{\pi}{2}$ D) $\frac{4+\pi}{2}$

3) $\int_0^{\log 5} \frac{e^x \sqrt{e^x-1}}{e^x+3} dx =$

A) $3 + 2\pi$ B) $4 - \pi$ C) $2 + \pi$ D) $4 + \pi$

4) $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^2 x dx =$

A) $\frac{7\pi}{256}$ B) $\frac{3\pi}{256}$ C) $\frac{5\pi}{256}$ D) $\frac{-5\pi}{256}$

5) If $\int_0^1 \frac{dx}{\sqrt{1+x-\sqrt{x}}} = \frac{k}{3}$, then k is equal to

A) $\sqrt{2}(2\sqrt{2}-2)$ B) $\frac{\sqrt{2}}{3}(2-2\sqrt{2})$
C) $\frac{2\sqrt{2}-2}{3}$ D) $4\sqrt{2}$

6) $\int_1^2 \frac{1}{x^2} e^{\frac{1}{x}} dx =$

A) $\sqrt{e} + 1$ B) $\sqrt{e} - 1$ C) $\sqrt{e}(\sqrt{e} - 1)$ D) $\frac{\sqrt{e}-1}{e}$

7) If $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx = a + \frac{b}{\log 2}$, then

A) $a = e, b = -2$ B) $a = e, b = 2$
C) $a = -e, b = 2$ D) $a = -e, b = -2$

8) Let $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then

A) $I_1 = \frac{1}{3} I_2$ B) $I_1 + I_2 = 0$
C) $I_1 = 2I_2$ D) $I_1 = I_2$

9) $\int_0^9 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} dx =$

A) 9 B) $\frac{9}{2}$ C) 0 D) 1

10) The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2+\sin \theta}{2-\sin \theta}\right) d\theta$ is

A) 0

B) 1

C) 2

D) π **Answers to MCQ's**

1	A	3	B	5	D	7	A	9	B
2	A	4	C	6	C	8	D	10	A

II) Evaluate the following:

1) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3 \cdot \cos x + \sin x} dx$

Solution: Let $I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{3 \cos x + \sin x} dx$

Let $\cos x = A(3 \cos x + \sin x) + B \frac{d}{dx}(3 \cos x + \sin x)$

$\therefore \cos x = A(3 \cos x + \sin x) + B(-3 \sin x + \cos x)$

$\therefore \cos x = (3A + B) \cos x + (A - 3B) \sin x$

Comparing the coefficients of $\cos x$ and $\sin x$, we get

$3A + B = 1, A - 3B = 0$

By solving these equations, we get

$A = \frac{3}{10}$ and $B = \frac{1}{10}$

$\therefore \cos x = \frac{3}{10}(3 \cos x + \sin x) + \frac{1}{10}(-3 \sin x + \cos x)$

$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\frac{3}{10}(3 \cos x + \sin x) + \frac{1}{10}(-3 \sin x + \cos x)}{3 \cos x + \sin x} dx$

$= \frac{3}{10} \int_0^{\frac{\pi}{2}} 1 dx + \frac{1}{10} \int_0^{\frac{\pi}{2}} \frac{-3 \sin x + \cos x}{3 \cos x + \sin x} dx$

$= \frac{3}{10} [x]_0^{\pi/2} + \frac{1}{10} [\log|3 \cos x + \sin x|]_0^{\pi/2}$

$\dots \left[\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$

$= \frac{3}{10} \left(\frac{\pi}{2} - 0 \right) + \frac{1}{10} [\log|3 \cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})|$

$- \log|3 \cos 0 + \sin 0|]$

$= \frac{3\pi}{20} + \frac{1}{10} [\log(3 \times 0 + 1) - \log(3 \times 1 + 0)]$

$= \frac{3\pi}{20} + \frac{1}{10} (\log 1 - \log 3)$

$= \frac{3\pi}{20} + \frac{1}{10} (0 - \log 3)$

$\therefore I = \frac{3\pi}{20} - \frac{1}{10} \log 3$

विद्यार्थी मित्रांनो वरील example मध्ये प्रथमतः N ला A(D) + B $\frac{d}{dx}$ (D) या form मध्ये लिहून co-efficient compare करून A व B यांच्या Value काढणे. व उदा. दाखवल्याप्रमाणे simplification करून integration formula चा वापर करून व limit put करून integration solve करावे.

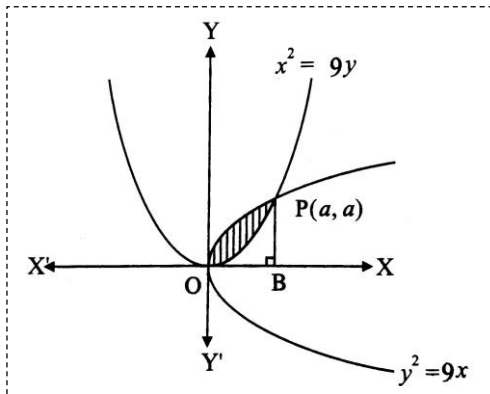
Solved Examples:

Ex. 1) Find the area of the region bounded by the curves $y^2 = 9x$ and $x^2 = 9y$.

Solution: The equations of the curves are

$$y^2 = 9x \dots\dots (I)$$

$$\text{and } x^2 = 9y \dots\dots (II)$$



Squaring equation (II)

$$x^4 = 81y^2 \quad x^4 = 81(9x) \dots \text{by (1)}$$

$$x^4 = 729x \quad \therefore x(x^3 - 9^3) = 0$$

$$\text{i.e. } x(x^3 - 9^3) = 0$$

$$\Rightarrow x = 0 \quad \text{or } x = 9$$

From equation (II), $y = 0$ or $y = 9$

$\therefore$ The points of intersection of the curves are $(0, 0)$, $(9, 9)$.

$$\therefore \text{ Required area } A = \int_0^9 \sqrt{9x} \, dx - \int_0^9 \frac{x^2}{9} \, dx$$

$$= \left[3 \cdot \frac{2}{3} \cdot x^{\frac{3}{2}} \right]_0^9 - \left[\frac{1}{9} \cdot \frac{x^3}{3} \right]_0^9$$

$$= 2 \cdot 9^{\frac{3}{2}} - 27$$

$$A = 54 - 27$$

$$= 27 \text{ sq. units}$$

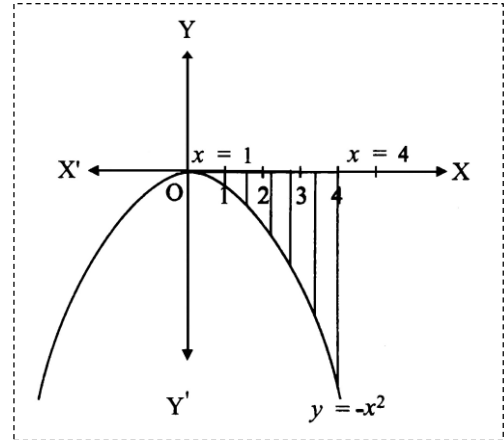
Now, we will see how to find the area bounded by the curve $y = f(x)$, X -axis and lines $x = a$, $x = b$ if $f(x)$ is negative i.e. $f(x) \leq 0$ in $[a, b]$

विद्यार्थी मित्रांनो, या example मध्ये प्रथमतः दिलेल्या curve च्या equation पासून उदा. दाखविल्याप्रमाणे diagram काढून घेणे नंतर उदा. दाखविल्याप्रमाणे त्याचे simplification करून x व y च्या value काढून घेणे व

$A = \left| \int_{x=a}^{x=b} f(x) dx - \int_{x=a}^{x=b} g(x) dx \right|$ या formula चा वापर करून required area काढणे.

Ex.2) Find the area bounded by the curve $y = -x^2$, X -axis and lines $x = 1$ and $x = 4$.

Solution: Let A be the area bounded by the curve $y = -x^2$, X -axis and $1 \leq x \leq 4$.



The required area A

$$= \int_1^4 y \, dx$$

$$= \int_1^4 -x^2 \, dx$$

$$= \left[-\frac{x^3}{3} \right]_1^4$$

$$= -\frac{64}{3} + \frac{1}{3}$$

$$A = -21,$$

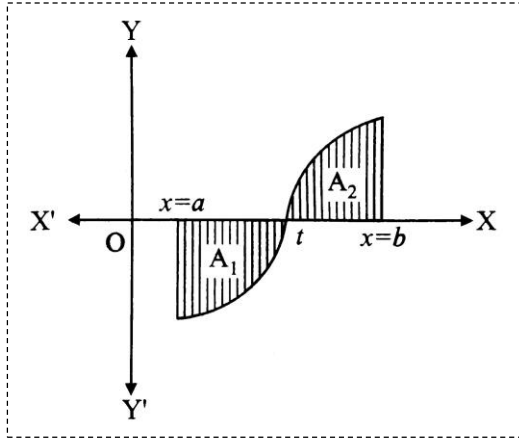
But we consider the area to be positive.

$$\therefore A = |-21| \text{ sq. units} = 21 \text{ square units.}$$

Thus, if $f(x) \leq 0$ or $f(x) \geq 0$ in $[a, b]$ then the area enclosed between $y = f(x)$, X -axis and $x = a$, $x = b$ is $\left| \int_a^b f(x) \cdot dx \right|$.

या example मध्ये $y = -x^2$ या curve साठी उदा. दाखविल्याप्रमाणे diagram काढणे A हा x -axis ला $x = 1$ आणि $x = 4$ या line मध्ये bound करणाऱ्या $y = -f(x)$ या curve चा area आहे. तर $A = \int_1^4 y \, dx$ चा वापर करून उदा. दाखविल्याप्रमाणे area काढणे जर आपण area हा positive consider केला तर $y = f(x)$, X -axis आणि $x = a$, $x = b$

असणाऱ्या curve चा area हा $\left| \int_a^b f(x) \cdot dx \right|$ हा आहे.



If the area A is divided into two parts A_1 and A_2 such that A_1 is the part of $a \leq x \leq t$ where $f(x) \leq 0$ and A_2 is the part of $a \leq x \leq t$ where $f(x) \geq 0$ then in A_1 the required area is below the X-axis and in A_2 , the required area is above the X-axis.

Now the total area $A = A_1 + A_2$

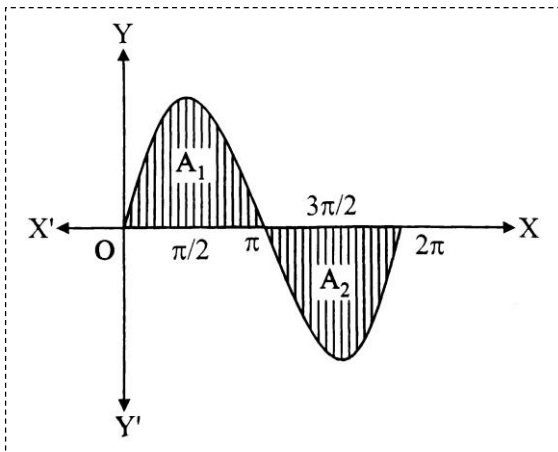
$$= \left| \int_a^t f(x) dx \right| + \left| \int_t^b f(x) dx \right|$$

Diagram मध्ये दाखविल्याप्रमाणे जर Area दोन part मध्ये divide होत असेल तर A_1 हा part $a \leq x \leq t$ ज्यात $f(x) \leq 0$ अस असेल A_2 हा part $a \leq x \leq t$ ज्यात $f(x) \geq 0$ अस असेल A_1 हा area X-axis च्या खाली तर A_2 हा area X-axis च्या वर lie करेल. तर total area

$$\text{हा } A = A_1 + A_2 = \left| \int_a^t f(x) dx \right| + \left| \int_t^b f(x) dx \right| \text{ असा असेल.}$$

Ex. 3: Find the area enclosed between the X-axis and the curve $y = \sin x$ for values of x between 0 to 2π .

Solution: The area enclosed between the curve and the X-axis consists of equal area lying alternatively above and below X-axis which are respectively positive and negative.



$$\begin{aligned} 1) \text{ Area } A_1 &= \text{area lying above the X-axis} \\ &= \int_0^\pi \sin x \, dx = [-\cos x]_0^\pi \\ &= -[\cos \pi - \cos 0] = -(-1, -1) \\ A_1 &= 2 \end{aligned}$$

$$\begin{aligned} 2) \text{ Area } A_2 &= \text{area lying below the X-axis} = \int_\pi^{2\pi} \sin x \, dx = \\ &= [-\cos x]_\pi^{2\pi} = [-\cos 2\pi - \cos \pi] \\ &= -[-1 - (-1)] \\ A_2 &= -2 \end{aligned}$$

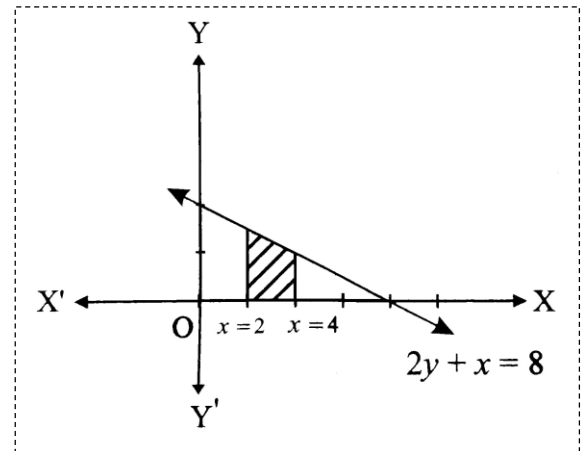
$$\therefore \text{ Total area} = A_1 + |A_2| = 2 + |(-2)| = 4 \text{ sq. units.}$$

X-axis आणि curve मध्ये enclosed होणारा area हा X-axis च्या खाली आणि वर equal part मध्ये divide होतो जो की अनुक्रमे positive आणि negative असा आहे, A_1 हा X-axis च्या above lie करणारा Area तर A_2 हा area X-axis च्या below lie करणारा Area तर $A_1 = \int_0^\pi \sin x \, dx$ व $A_2 = \int_\pi^{2\pi} \sin x \, dx$ अशा पद्धतीने उदा. दाखविल्याप्रमाणे simplify करून $A = A_1 + |A_2|$ Total area काढणे.

Ex.4): Using integration, find the area of the region bounded by the line $2y + x = 8$, X-axis and the lines $x = 2$ and $x = 4$.

Solution: The required region is bounded by the lines $2y + x = 8$, and $x = 2$, $x = 4$ and X-axis.

$$\therefore y = \frac{1}{2} (8 - x) \text{ and the limits are } x = 2, x = 4.$$



TOPIC - 07

PROBABILITY DISTRIBUTIONS

Basic Terminology

1) Random experiment (trial):

Any action in which all the possible results are known in advance, but none of them can be predicted is called a random experiment.

2) Sample space:

The set of all possible outcomes of a random experiment is called the sample space of that experiment and is denoted by S .

Every element of the sample space is called the sample point. The number of sample points in a sample space S are denoted by $n(S)$.

3) Event:

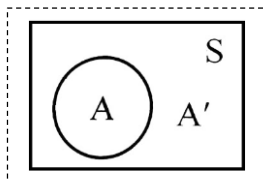
An event is a subset of the sample space and is usually denoted by capital letters A, B, C , etc.

4) Equally likely outcomes:

The outcomes of a random experiment are called equally likely, if all of them have equal preference. In the experiment of tossing an unbiased coin, the outcomes 'Head' and 'Tail' are equally likely.

5) Complement of an event:

The set of all outcomes in a sample space excluding those of an event is called the complement of an event. It is denoted by A', B', C' , etc.



Using a Venn Diagram we represent the Sample Space (S), an Event (A), the Complement of an Event (A') etc., as shown above.

Some commonly used events and their notations are given below:

Event	Notation
Not A	$\bar{A}$ or A' or A^c
at least one of A, B occurs	$A \cup B$
both A and B occur	$A \cap B$
A occurs but not B	$A \cap B'$
B occurs but not A	$A' \cap B$
neither A nor B occur	$A' \cap B'$
at least one of A, B, C	$A \cup B \cup C$
Exactly one of A and B	$(A \cap \bar{B}) \cup (\bar{A} \cap B)$
All three of A, B, C	$A \cap B \cap C$
Exactly two of A, B and C	$(A \cap B \cap \bar{C}) \cup$ $(A \cap \bar{B} \cap C) \cup$ $(\bar{A} \cap B \cap C)$

विद्यार्थ्यांनी वर दिलेल्या सर्व event आणि Notation चा अभ्यास व्यवस्थित करावा.

6) Exhaustive events:

Two events A and B defined on a sample space S are said to be exhaustive if $A \cup B = S$.

7) Mutually Exclusive events:

Two events A and B defined on a sample space S are called mutually exclusive events if they can not occur simultaneously, i.e., $A \cap B = \phi$. In such a case A and B are disjoint sets.

Note: If two events A and B defined on a sample space S are mutually exclusive and exhaustive, then they are called complementary events.

That is, if $A \cup B = S$ and $A \cap B = \phi$ then A and B are called complementary events.

8) Tossing a coin/coins:

If one coin is tossed n times or n coins are tossed together, then $n(S) = 2^n$

9) Throwing a die/dice:

If one die is thrown n times or n dice are thrown together, then $n(S) = 6^n$

10) Playing cards:

A full pack of playing cards, also called the deck, consists of 52 cards.

It does not contain any joker.

Amongst these 52 cards, there are 4 different suits, each containing 13 cards.

Of these four suits, 2 are red and 2 are black.

The two red suits are the hearts ♥ and the diamonds (♦).

The two black suits are the spades (♠) and the clubs (♣).

In each suit, each of the thirteen cards is assigned a definite number, from 1 to 13, called its denomination.

In each suit, there are 3 picture (face) cards, viz. a jack, a queen and a king.

The card with denomination 1 is called an ace.

1) Random experiment (trial):

कोणतीही कृती ज्यामध्ये सर्व संभाव्य परिणाम अगोदरच माहिती असतात. पण त्यापैकी आपण काहीही सांगू शकत नाही अशा कृतीला आपण random experiment असे म्हणतो.

2) Sample space:

Random Experiment मधील सर्व संभाव्य परिणामाच्या (All possible outcomes) set (संच) ला आपण sample space असे म्हणतो. तो s ने दर्शविला जातो. Sample space मधील प्रत्येक element ला sample point म्हणटले जाते. व ते $n(s)$ ने दर्शविला जाते.

3) Event:

Sample space च्या subset ला event असे म्हणटले जाते आणि ते capital letter A, B, C, etc ने दर्शविला जाते.

4) Equally likely outcomes:

एखाद्या random experiment मध्ये सर्व संभाव्य परिणाम हे जर आपल्याला equal preference मध्ये मिळत असेल तर त्याला equally likely outcomes असे म्हणतात.

5) Complement of an event:

Sample space मधील असे element जे event मध्ये नाहीत त्याला आपण त्या set चा complement असे म्हणतो जो A', B', C' element असा दर्शविला जातो.

6) Exhaustive events:

जर A व B या दोन set चे union केले असता आपल्याला sample spaces जर मिळत असेल तर त्याला आपण exhaustive events म्हणतो.

7) Mutually Exclusive events:

जर A व B या दोन set चे intersection केले असता आपल्याला कोणताही element मिळत नसेल म्हणजेच ϕ मिळत असेल तर त्याला आपण mutually exclusive event असे म्हणतो अशा cases मध्ये A व B हे disjoint set असतात.

Note:

जर दोन event A व B हे दोन्ही mutually exclusive व exhaustive असतील तर त्या event ला आपण complementary event असे म्हणतो.

8) Tossing a coin/coins:

जर एक coin n time tossed केला किंवा n coins tossed केले तर त्याची संभाव्यता $n(s) = 2^n$ ही आहे.

9) Throwing a die/dice:

जर एक die n time thrown केला किंवा n die एकत्र thrown केले तर त्याची संभाव्यता $n(s) = 6^n$ ही आहे.

10) Playing cards:

विद्यार्थी मित्रांनो आपण सर्वानी पत्ते पाहिले असतील त्यालाच आपण pack of playing card म्हणजेच deck म्हणतो जे की ५२ cards ने बनते. जर त्यात joker नसेल तर एकूण ५२ card मध्ये चार parts पडतात त्या प्रत्येक मध्ये १३ cards असतात. एकूण ४ part मधील २ part हे red आणि २ part हे Black असतात. त्या दोन red part मध्ये एक heart (बदाम), एक diamond (चौकट) असतो तर दोन Black part मध्ये एक spade (एस्पीक) आणि एक clubs (क्लब) असतो.

प्रत्येक part मधील १३ card हे १ ते १३ य number defined केले आहेत त्यांना denomination असे म्हणटले जाते. प्रत्येक part मध्ये १३ card, त्यामध्ये तीन face असलेले पत्ते असतात त्यांना Jack, queen आणि king म्हणतात. card ज्याच denomination one आहे त्याला ace असे म्हणतात.

Exercise 7.1

1) Let X represents the difference between number of heads and number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Solution: X represent the difference between number of heads and number of tails.

Sample space of the experiment is.

$S = \{(0 \text{ heads, } 6 \text{ tails}), (1 \text{ head, } 5 \text{ tails}), (2 \text{ heads, } 4 \text{ tails}), (3 \text{ heads, } 3 \text{ tails}), (4 \text{ heads, } 2 \text{ tails}), (5 \text{ heads, } 1 \text{ tail}), (6 \text{ heads, } 0 \text{ tails})\}$

The value of X corresponding to these outcomes are as follow:

$$X(0 \text{ heads, } 6 \text{ tails}) = 0 - 6 = -6$$

$$X(1 \text{ heads, } 5 \text{ tails}) = 1 - 5 = -4$$

$$X(2 \text{ heads, } 4 \text{ tails}) = 2 - 4 = -2$$

$$X(3 \text{ heads, } 3 \text{ tails}) = 3 - 3 = 0$$

$$X(4 \text{ heads, } 2 \text{ tails}) = 4 - 2 = 2$$

$$X(5 \text{ heads, } 1 \text{ tails}) = 5 - 1 = 4$$

$$X(6 \text{ heads, } 0 \text{ tails}) = 6 - 0 = 6$$

$\therefore$ Possible values of X are $\{-6, -4, -2, 0, 2, 4, 6\}$

या example मध्ये x हा number of heads आणि tails मधील difference represent करतोय आणि त्यासाठी सुरवातीला आपण sample space लिहून घेतला. जसे की 0 heads असतील तर आपल्याला 6 tails मिळेल. अशा प्रकारे आपण sample space लिहिला आणि प्रत्येक sample point साठी उदाहरणात दाखविल्याप्रमाणे simplification करून possible value of x काढून घेतल्या.

2) State which of the following are not the probability mass function of a random variable. Give reasons for your answer.

i)

i) X	0	1	2
P(X)	0.4	0.4	0.2

ii)

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.2

iii)

X	0	1	2
P(X)	0.1	0.6	0.3

iv)

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0	0.05

v)

Y	-1	0	1
P(Y)	0.6	0.1	0.2

vi)

X	0	-1	-2
P(X)	0.3	0.4	0.3

Solution: i) Here, $p_i > 0, \forall i = 1, 2, 3$

Now consider,

$$\sum_{i=1}^3 p_i = 0.4 + 0.4 + 0.2 = 1$$

$\therefore$ Given distribution is p.m.f.

ii) Here, $P(X = 3) = -0.1 < 0$

$\therefore$ Given distribution is not p.m.f.

iii) Here, $p_i > 0, \forall i = 1, 2, 3$

Now consider,

$$\sum_{i=1}^3 p_i = 0.1 + 0.6 + 0.3 = 1$$

$\therefore$ Given distribution is p.m.f.

iv) Here, $p_i \geq 0, \forall i = 1, 2, \dots, 5$

Now consider,

$$\sum_{i=1}^5 p_i = 0.3 + 0.2 + 0.4 + 0 + 0.05 = 0.95 \neq 1$$

$\therefore$ Given distribution is not p.m.f.

v) Here, $p_i > 0, \forall i = 1, 2, 3$

Now consider,

$$\sum_{i=1}^3 p_i = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$$

$\therefore$ Given distribution is not p.m.f.

vi) Here, $p_i > 0, \forall i = 1, 2, 3$

Now consider,

$$\sum_{i=1}^3 p_i = 0.3 + 0.4 + 0.3 = 1$$

$\therefore$ Given distribution is p.m.f.

वरील सर्व example मध्ये समजा p_i ची value ही 0 पेक्षा मोठी असेल तर दिलेले distribution हे p.m.f. आहे असे आपण म्हणतो.

Multiple Choice Questions:**1) A. p. m. f. of r.v.X is given below:**

X = x	2	4	6
P(X = x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{2}$

Then E(X) =

- (A) 5.25 (B) 4.25 (C) 4.75 (D) 5.75

2) A r.v.X has the following probability distribution

X	-1	-2	1	2
P(X)	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Then E(X) =

- A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{6}$

3) Number of stars in the sky is an example of

- A) continuous r. v.
 B) discrete r. v.
 C) both continuous and discrete r. v.
 D) neither continuous nor discrete.

4) The total probability of all the exhaustive events is

- A) 0 B) 1 C) -1 D) ± 1

5) Atmospheric pressure at a certain place in $\frac{N}{m^2}$ is

- A) continuous r. v.
 B) discrete r. v.
 C) both continuous and discrete r. v.
 D) neither continuous nor discrete.

6) If $P(x) = \frac{kx}{3}$, $x = 1, 2, 3$ is p.m.f. of X, then k =

- A) $\frac{1}{3}$ B) $\frac{1}{4}$ C) $\frac{1}{5}$ D) $\frac{1}{2}$

7) A die is thrown at random, what is the expectation of the number on it?

- A) 3.5 B) 3.6 C) 4.5 D) 4.6

8) If a pair of dice is thrown and X denotes the sum of the numbers on them. Find the expectation of X.

- A) 5 B) 0 C) 1 D) 7

9) For the given probability distribution, find E(X)

X	1	2	3
---	---	---	---

P(X = x)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{5}{8}$
----------	---------------	---------------	---------------

- A) $\frac{11}{8}$ B) $\frac{17}{8}$ C) $\frac{19}{8}$ D) $\frac{21}{8}$

10) In a game, a person is paid Rs.5 if he gets all heads or all tails when three coins are tossed and he will pay Rs.3 if either one or two heads show. What can he expect on an average per game?

- A) gain of ₹ 2 B) gain of ₹ 1
 C) loss of ₹ 1 D) No loss no gain

11) A random variable X has the following probability distribution values of X,

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	3k	k^2	$2k^2$	$2k^2$	$7k^2 + k$

Find value of $P(X \geq 6)$.

- A) $\frac{13}{100}$ B) $\frac{17}{100}$ C) $\frac{21}{100}$ D) $\frac{19}{100}$

12) The probability distribution of a discrete random variable X is given below:

X	2	3	4	5
P(X)	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

Then k =

- A) 8 B) 32 C) 16 D) 48

13) Let X be a random variable. The probability distribution of X is given below:

X	30	10	-10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E(X) =

- A) 6 B) -5 C) 4 D) -3

14) Let X be a discrete random variable assuming values $x_1, x_2, x_3, \dots, x_n$ with respective probabilities $p_1, p_2, p_3, \dots, p_n$. Then variance of X is given by

- A) $E(x^2)$ B) $E(x^2) + E(X)$
 C) $E(X^2) - \sqrt{E(X)}$ D) $E(X^2) - [E(X)]^2$

II) Solve the following**1) Find the principal solutions of the following equations:**

i) $\sin 2\theta = -\frac{1}{2}$ ii) $\tan 3\theta = -1$ iii) $\cot \theta = 0$

Solution: i) $\sin 2\theta = -\frac{1}{2}$

$$\sin 2\theta = -\sin \frac{\pi}{6}$$

$$\sin 2\theta = \sin \left(\pi + \frac{\pi}{6} \right) \quad \dots [\because \sin(\pi + \theta) = -\sin \theta]$$

$$\sin 2\theta = \sin \frac{7\pi}{6}$$

$$\text{Since } \sin \theta = \sin \alpha \text{ implies } \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$$

$$\therefore 2\theta = n\pi + (-1)^n \frac{7\pi}{6} \quad \therefore \theta = \frac{n\pi}{2} + (-1)^n \frac{7\pi}{12}$$

Values of n	Values of θ
0	$\frac{7\pi}{12}$
1	$\frac{-\pi}{12}$
2	$\frac{19\pi}{12}$
3	$\frac{11\pi}{12}$
4	$\frac{31\pi}{12}$
5	$\frac{23\pi}{12}$

Since $0 \leq \theta < 2\pi$, the required principal solutions are

$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}.$$

ii) $\tan 3\theta = -1$

Sol: Home work

iii) $\cot \theta = 0$

$$\therefore \cot \theta = \cot \left(\frac{\pi}{2} \right) \text{ and } \cot \theta = \cot \left(\pi + \frac{\pi}{2} \right) = \cot \left(\frac{3\pi}{2} \right)$$

$$\text{such that } 0 < \frac{\pi}{2} < 2\pi \text{ and } 0 < \frac{3\pi}{2} < 2\pi$$

$$\therefore \text{The required principal solutions are } \theta = \frac{\pi}{2} \text{ and } \theta = \frac{3\pi}{2}$$

विद्यार्थी मित्रांनो वरील सर्व example हे Ex ३.१ मध्ये सोडवलेल्या example चा अभ्यास करून सोडवणे.

2) Find the principal solutions of the following equations:

i) $\sin 2\theta = -\frac{1}{\sqrt{2}}$ ii) $\tan 5\theta = -1$ iii) $\cot 2\theta = 0$

Solution: i) $\sin 2\theta = -\frac{1}{\sqrt{2}}$

$$\sin 2\theta = -\sin \frac{\pi}{4}, \quad \sin 2\theta = \sin \frac{5\pi}{4}$$

$$\text{Since } \sin \theta = \sin \alpha \text{ implies } \theta = n\pi + (-1)^n \alpha$$

$$\therefore 2\theta = n\pi + (-1)^n \frac{5\pi}{4} \quad \therefore \theta = \frac{n\pi}{2} + (-1)^n \frac{5\pi}{8}$$

Values of n	Values of θ
0	$\frac{5\pi}{8}$
1	$\frac{-\pi}{8}$
2	$\frac{13\pi}{8}$
3	$\frac{7\pi}{8}$
4	$\frac{21\pi}{8}$
5	$\frac{15\pi}{8}$

Since $0 \leq \theta < 2\pi$, the required principal solutions are

$$\theta = \frac{5\pi}{8}, \frac{13\pi}{8}, \frac{7\pi}{8}, \frac{15\pi}{8}.$$

ii) $\tan 5\theta = -1$

$$\tan 5\theta = -\tan \frac{\pi}{4} = \tan \left(\frac{3\pi}{4} \right)$$

$$\text{Since } \tan \theta = \tan \alpha \text{ implies } \theta = n\pi + \alpha$$

$$\therefore 5\theta = n\pi + \frac{3\pi}{4} \quad \therefore \theta = \frac{n\pi}{5} + \frac{3\pi}{20}$$

Values of n	Values of θ
0	$\frac{3\pi}{20}$
1	$\frac{7\pi}{20}$
2	$\frac{11\pi}{20}$
3	$\frac{15\pi}{20}$
4	$\frac{19\pi}{20}$
5	$\frac{23\pi}{20}$
6	$\frac{27\pi}{20}$

Distance between skew lines:**Theorem:**

The distance between lines $\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$ and

$$\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2 \text{ is } \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

Proof:

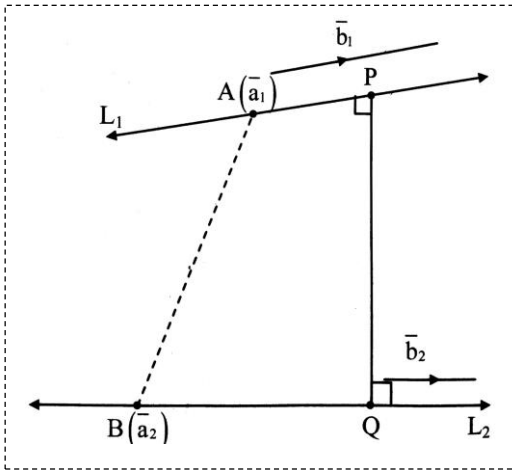
Let L_1 and L_2 be the lines whose equations are $\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$, $\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2$ respectively. Let PQ be the segment which is perpendicular to both L_1 and L_2 .

To find the length of segment PQ.

Lines L_1 and L_2 pass through points $A(\vec{a}_1)$ and $B(\vec{a}_2)$ respectively.

Lines L_1 and L_2 are parallel to $\vec{b}_1$ and $\vec{b}_2$ respectively.

As PQ is perpendicular to both L_1 and L_2 , it is parallel to $\vec{b}_1 \times \vec{b}_2$



The unit vector along $\overline{PQ}$ = unit vector along $\vec{b}_1 \times \vec{b}_2 = \hat{n}$ (say)

PQ = The projection of $\overline{AB}$ on $\overline{PQ} = \overline{AB} \cdot \hat{n}$

$$PQ = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

The shortest distance between lines

$\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2$ is

$$\left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$ आणि $\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2$ या line मधील distance हे $\left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot \vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} \right|$ हे आहे.

Remark:

Two lines intersect each other if and only if the shortest distance between them is zero.

Lines $\vec{r} = \vec{a}_1 + \lambda_1 \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda_2 \vec{b}_2$ intersect each other if and only if $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

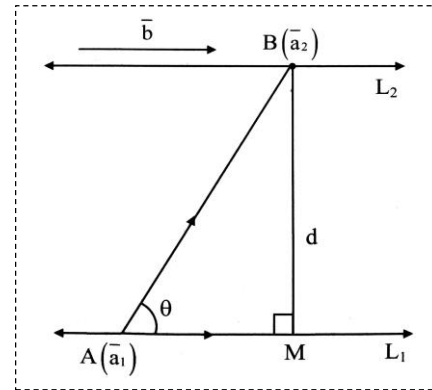
Lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ intersect each other if and only if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

दोन line एकमेकांना intersect केव्हा करतील जेव्हा त्यामधील shortest distance हे zero असेल आणि हे vice versa prove होते.

Distance between parallel lines:**Theorem 6.7**

The distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}$ is $|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$

Proof:

Let lines represented by $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}$ be l_1 and l_2

Lines L_1 and L_2 pass through $A(\vec{a}_1)$ and $B(\vec{a}_2)$ respectively.

Let BM be perpendicular to L_1 . To find BM

ΔAMB is a right angle triangle. Let $\angle BAM = \theta$

$$\sin \theta = \frac{BM}{AB}$$

$$\therefore BM = AB \sin \theta = AB \cdot |\vec{b}| \cdot \sin \theta = |\vec{AB} \times \vec{b}| = |(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$$

$\therefore$ The distance between parallel lines.

$\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}$ is given by

$$d = BM = |(\vec{a}_2 - \vec{a}_1) \times \vec{b}|$$

A linear equation in two variables $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$ and a and b are not zero simultaneously represents a straight line. A straight line makes disjoint parts of the plane. The points lying on the straight line and two half planes on either side, which are represented by $ax + by + c < 0$ and $ax + by + c > 0$. We will now study the two half planes made by a line.

The sets of points $\{(x, y) | ax + by + c < 0\}$ and $\{(x, y) | ax + by + c \geq 0\}$ are two open half planes. The sets of points $\{(x, y) | ax + by + c \leq 0\}$ and $\{(x, y) | ax + by + c \geq 0\}$ are half planes with common points. The sets $\{(x, y) | ax + by + c \leq 0\}$ and $\{(x, y) | ax + by + c \geq 0\}$ have the common boundary $\{(x, y) | ax + by + c = 0\}$.

7.1 Linear inequation in two variables:

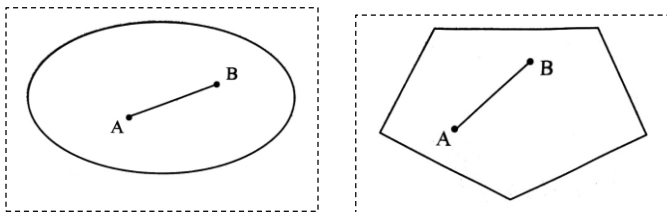
A linear inequation in two variables x, y a mathematical expression of the form $ax + by < c$ or $ax + by > c$, where $a \neq 0, b \neq 0$ simultaneously and $a, b \in \mathbb{R}$.

विद्यार्थी मित्रांनो, $ax + by < c$ or $ax + by > c$ अशा form मध्ये असणाऱ्या mathematical expression ला आपण linear equation in two variable अस म्हणतो. x, y हे दोन variable आहेत आणि a, b हे दोन्ही non-zero real number आहेत.

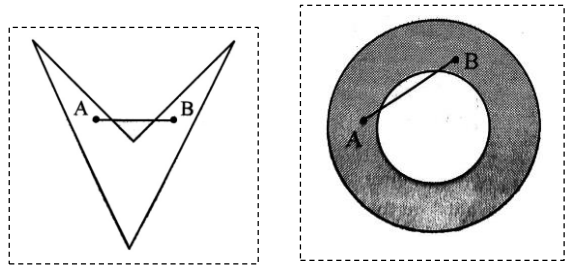
Convex set: A set of points in a plane is said to be a convex set if line segment joining any two points of the set entirely lies within the set.

Plane मध्ये set of point असून त्यातील दोन points पासून line segment काढली असता ती पूर्णपणे set मध्ये lies होते.

The following sets are convex sets:

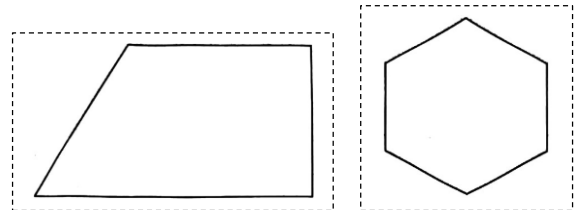


The following sets are not convex sets.

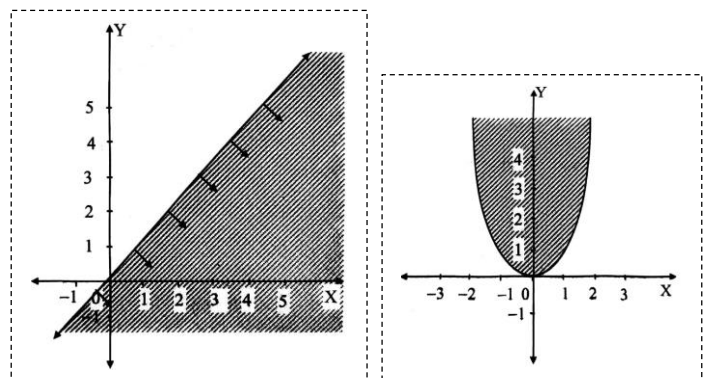


Note:

i) The convex sets may be bounded. Following are bounded convex sets.



ii) Convex sets may be unbounded.



Convex set हे may be bounded (बद्ध) (मर्यादित) असतात. अशा set ला आपण convex set असे म्हणतो.

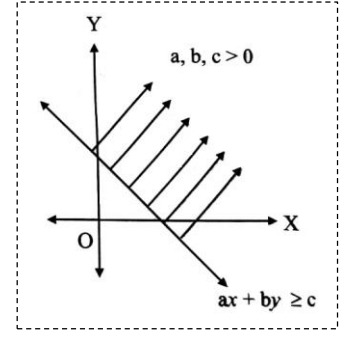
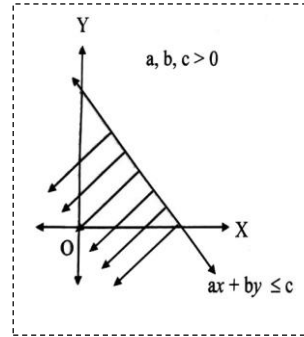
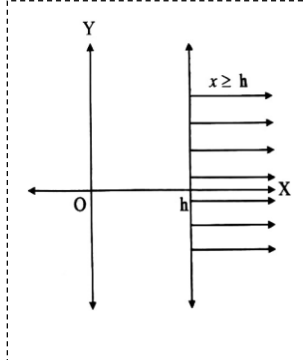
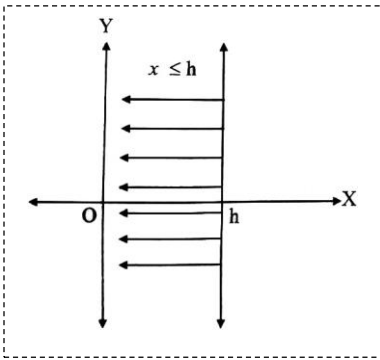
Convex set हे may be unbounded (अमर्यादित) असतात.

Note:

i) Graphical representation of $x \leq h$ and $x \geq h$ on the Cartesian co-ordinate system.

Draw the line $x = h$ in XOY plane.

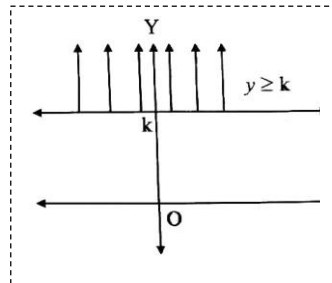
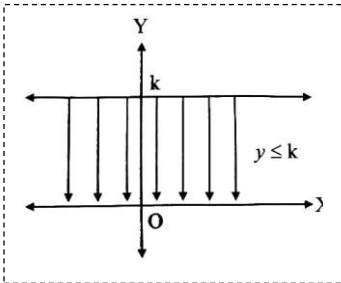
The solution set in the set of points lying on Left side or Right side of the line $x = h$



ii) Graphical representation of $y \leq k$ and $y \geq k$ on the Cartesian co-ordinate system.

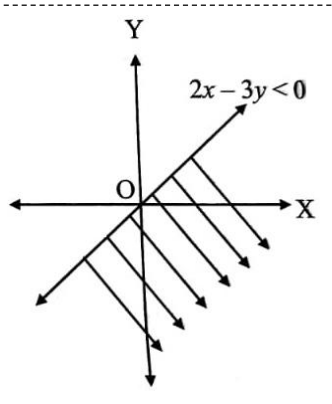
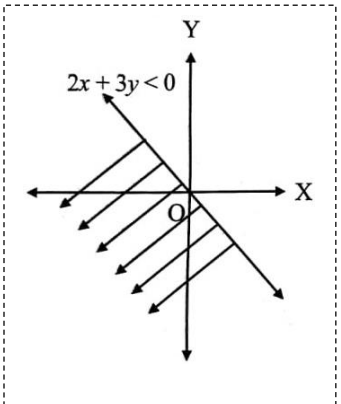
Draw the line $y = k$ in XOY plane.

The solution set is the set of points lying below or above the line $y = k$.



iii) Graphical representation of $ax + by \leq 0$ and $ax + by \geq 0$ on the Cartesian co-ordinate system.

The line $ax + by = 0$ passes through the origin, see the following graphs.



iv) To find the solutions of $ax + by \leq c$ and $ax + by \geq c$ graphically,

Draw the line $ax + by = c$ in XOY system. It divides the plane into two parts, each part is called half plane:

The half plane H_1 containing origin is called origin side of the line and the other half plane H_2 is non-origin side of the line $ax + by = c$.

For convenience, consider $O(0, 0)$ as test point. If $O(0, 0)$ satisfies the given inequality $ax + by \leq c$, then the required region is on origin side. Hence shade the region H_1 otherwise shade the other half planes. H_2 .

The shaded portion represents the solution set of the given inequality.

Note that the points $\{(x, y) \mid ax + by = c\}$ form the common boundary of the two half planes.

१) $x \leq h$ आणि $x \geq h$ या cartesian co-ordinate system चे graphical representation - XOY plane मध्ये $x = h$ या line च्या डाव्या बाजूला आपल्याला solution मिळेल कारण x हा h एवढा आणि h पेक्षा लहान आहे.

- $x \geq h$ साठी $x = h$ या line च्या उजव्या बाजूला आपल्याला solution मिळेल कारण x हा h एवढा आणि h पेक्षा मोठा आहे.

ii) $y \leq k$ आणि $y \geq k$ या cartesian co-ordinate system चे graphical representation

- XOY plane मध्ये $y = k$ ही line काढून घेणे.

- $y \leq k$ साठी $y = k$ या line च्या खालच्या बाजूला आपल्याला solution मिळेल कारण y हा k एवढा आणि k पेक्षा लहान आहे.

- $y \geq k$ साठी $y = k$ या line च्या वरच्या बाजूला आपल्याला solution मिळेल कारण y हा k एवढा आणि k पेक्षा मोठा आहे.

iii) $ax + by \leq 0$ आणि $ax + by \geq 0$ या cartesian co-ordinate system चे graphical representation $ax + by = 0$ ही line काढली असता ती origin मधून जाते.

iv) Solution of $ax + by \leq c$ आणि $ax + by \geq c$ graphically XOY या system मध्ये प्रथमतः $ax + by = c$ ही line घेतली ही line plane ला दोन part मध्ये divide करते. प्रत्येक part ला आपण half plane असे म्हणतो.

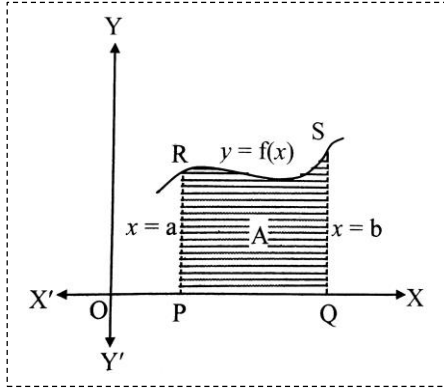
5.1 Area under the Curve

Area under a Curve:

i) The area 'A' bounded by the curve $y = f(x)$, X-axis and bounded between the lines $x = a$ and $x = b$ (see the figure below) is given by

$A = \text{Area of the region PRSQ}$

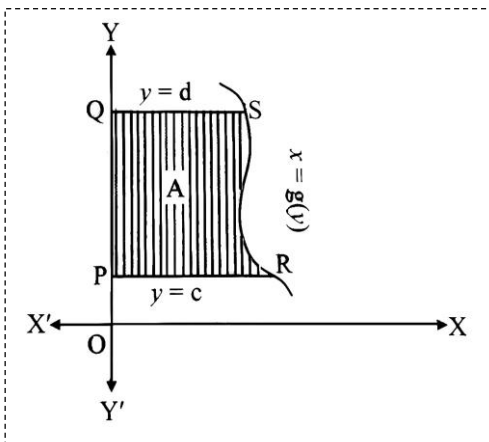
$$\therefore A = \int_a^b y \, dx = \int_{x=a}^{x=b} f(x) \, dx$$



ii) The area 'A' bounded by the curve $x = g(y)$ Y-axis and bounded between the lines $y = c$ and $y = d$ (see the figure below) is given by

$A = \text{Area of the region PRSQ}$

$$\therefore A = \int_c^d x \, dy = \int_{y=c}^{y=d} g(y) \, dy$$

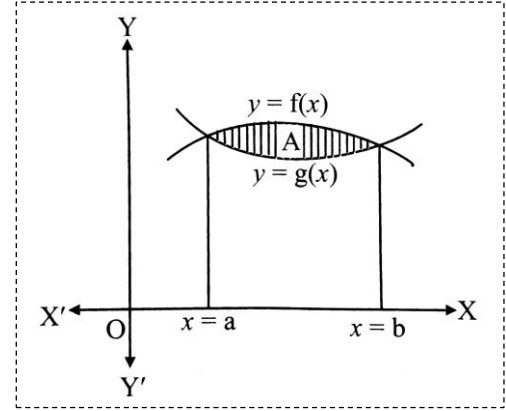


Area between two curves:

The area of the shaded region bounded by two curves $y = f(x)$ and $y = g(x)$ as shown in the figure is given by

$$A = \left| \int_{x=a}^{x=b} f(x) \, dx - \int_{x=a}^{x=b} g(x) \, dx \right|$$

Where the curves $y = f(x)$ and $y = g(x)$ intersect at the points $[a, f(a)]$ and $[b, f(b)]$.



Area under a Curve:

i) X-axis वर असणाऱ्या आणि $x=a$ आणि $x=b$ या line मध्ये region bound करणाऱ्या $y=f(x)$ या curve चा Area म्हणजेच diagram मध्ये दाखविल्याप्रमाणे,

Area of region PRSQ

$$\therefore A = \int_a^b y \, dx = \int_{x=a}^{x=b} f(x) \, dx \text{ हा आहे.}$$

ii) Y-axis वर असणाऱ्या आणि $y = c$ आणि $y = d$ या line मध्ये region bound करणाऱ्या $x = g(y)$ या curve चा Area म्हणजेच diagram मध्ये दाखविल्याप्रमाणे,

Area of region PRSQ

$$\therefore A = \int_c^d x \, dy = \int_{y=c}^{y=d} g(y) \, dy \text{ हा आहे.}$$

Area between two curves:

$y = f(x)$ आणि $y = g(x)$ या दोन curve पासून तयार होणारा Area हा diagram मध्ये shaded part मध्ये दाखवला आहे. त्या region चा area हा, $A = \left| \int_{x=a}^{x=b} f(x) \, dx - \int_{x=a}^{x=b} g(x) \, dx \right|$ हा आहे. curves $y = f(x)$ आणि $y = g(x)$ हे एकमेकांना $[a, f(a)]$ आणि $[b, f(b)]$ या point मध्ये intersect करतात.

अशाच प्रकारे ...

सर्व Topics च्या सर्व प्रश्नांची उत्तरे अगोदर English मध्ये दिली असून त्याचे स्पष्टीकरण Scientific मराठीमध्ये दिले आहे...

उभ्या महाराष्ट्रात असा एकही विद्यार्थी राहणार नाही की, त्याला समजणार नाही..

- ❖ सर्वांना सर्व समजणार...
- ❖ सर्वजण सर्व लिहिणार...
- ❖ मार्क्स/ ज्ञान सर्वांचे वाढणार...
- ❖ ज्ञान वाढल्याने बेरोजगारी कमी होणार...
- ❖ पाठांतर/ घोकमपट्टी करून काहीही उपयोग नाही...

यासाठी फक्त ज्ञानदाची पुस्तके

पुस्तक मागणीसाठी संपर्क - 9657961490

आजच संपर्क करा..



DNYANADA's

Upcoming Series of Guides

Std - 8th (Semi)

- English
- Mathematics
- General Science

Std - 9th (Semi)

- English
- Mathematics
- Science & Technology

Std - 10th (Semi)

- English
- Mathematics - I
- Mathematics - II
- Science & Technology - I
- Science & Technology - II

Std - 11th (Science) English & Scientific - मराठी

- Physics
- Chemistry
- Biology

Std - 12th (Science) CET - PYQ

- Physics
- Chemistry
- Biology
- Mathematics

Std - 12th (Science) English & Scientific - मराठी

- Physics
- Chemistry
- Biology
- Mathematics

Std - 12th (Science) - CET

- Physics
- Chemistry
- Biology
- Mathematics

श्री ज्ञानदा पब्लीकेशन हाऊस, पुणे
आता, MHT - CET ची पुस्तके
चक्क! दोन भाषेत... English व
Scientific मराठीमध्ये



श्री ज्ञानदा प्रकाशन,

गणराज बुक सेंटरच्या वरती, अप्पा बळवंत चौक, पुणे - 411002,
 Email : amoltarange1490@gmail.com 📞 9657961490

MRP:

450/-