

Strictly As Per
BOARDS New Syllabus

My Favorite Guide On **MATHEMATICS - II**

गणित भाग - 2

Std 10th - Semi English



DNYANADA PUBLICATION, PUNE

**महाराष्ट्रात
प्रथमच...**

11th आणि 12th (Science) ची पुस्तके
English व Scientific मराठी मधून...

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PUNE | LATUR PATTERN

TOPIC – 01 SIMILARITY

1.1 Introduction:

i) We have studied Ratio and Proportion. The statement, 'the numbers a and b are in the ratio $\frac{m}{n}$ ' is also written as, 'the numbers a and b are in proportion m:n.

ii) For this concept we consider positive real numbers. We know that the lengths of line segments and area of any figure are positive real numbers.

iii) We know the formula of area of a triangle.

Area of a triangle = $\frac{1}{2}$ Base $\times$ Height

Similarity (समरूपता)

1.1 Introduction (प्रस्तावना):

आपण Ratio and Proportion (गुणोत्तर व प्रमाण) यांचा अभ्यास केला आहे. a आणि b या दोन number चा ratio $\frac{m}{n}$ आहे, तेच statement (विधान) a आणि b या दोन numbers m:n या ratio मध्ये आहेत असेही लिहितात.

या concept साठी आपण general पणे positive real numbers (धन वास्तव संख्या) चा विचार करतो. आपल्याला हे information आहे की lengths of line segments (रेषाखंडाची लांबी) आणि area of any figure (एखाद्या आकृतीचे क्षेत्रफळ) या positive real numbers असतात.

आपल्याला area of a triangle (त्रिकोणाचे क्षेत्रफळ) चा formula माहीत आहे.

area of triangle = $\frac{1}{2}$ base $\times$ height

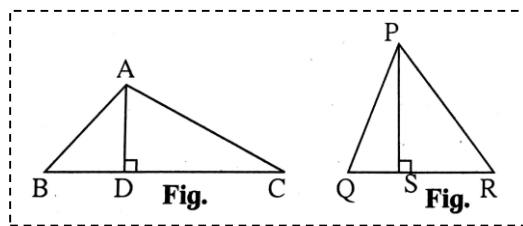
1.2 Ratio of areas of two triangles

Let's find the ratio of areas of any two triangles.

Ex. In ΔABC , AD is the height and BC is the base.

In ΔPQR , PS is the height and QR is the base

$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS} \therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$$



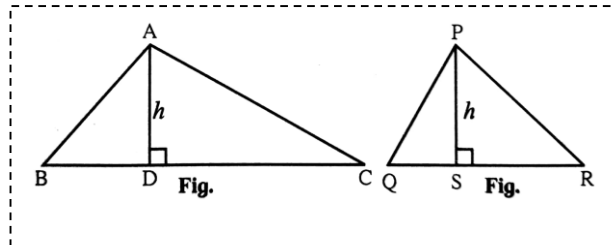
Hence the ratio of the areas of two triangles is equal to the ratio of the products of their bases and corresponding heights,

Base of a triangle is b_1 and height is h_1 . Base of another triangle is b_2 and height is h_2 , then the ratio of their areas = $\frac{b_1 \times h_1}{b_2 \times h_2}$.

Suppose some conditions are imposed on these two triangles,

Condition 1: If the heights of both triangles are equal then-

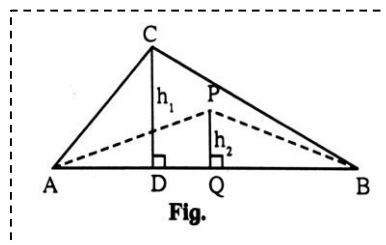
$$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC \times h}{QR \times h} = \frac{BC}{QR} \therefore \frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{b_1}{b_2}$$



Property: The ratio of the areas of two triangles with equal heights is equal to the ratio of their corresponding bases.

Condition 2: If the bases of both triangles are equal then

$$\frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{AB \times h_1}{AB \times h_2} \therefore \frac{A(\Delta ABC)}{A(\Delta APB)} = \frac{h_1}{h_2}$$



1.6 Similar triangles

1.6.1 Tests of similarity of triangles

- For similarity of two triangles, the necessary conditions are that their corresponding sides are in same proportion and their corresponding angles are congruent.
- Out of these conditions; when three specific conditions are fulfilled, the remaining conditions are automatically fulfilled.
- This means for similarity of two triangles, only three specific conditions are sufficient.
- Similarity of two triangles can be confirmed by testing these three conditions.
- The groups of such sufficient conditions are called tests of similarity, which we shall use.

Tests for similarity of triangles (त्रिकोणांच्या समरूपतेच्या कसोट्या)

दोन triangle similar असण्यासाठी त्यांच्या तिन्ही corresponding sides (संगत बाजू) प्रमाणात असणे आणि तिन्ही corresponding angle congruent (एकरूप) असणे आवश्यक असते; परंतु या सहा conditions पैकी three specific conditions ची fulfill (पूर्तता) झाल्यास उरलेल्या conditions fulfill आपोआप होते; म्हणजे दोन triangle similar होण्यासाठी three specific conditions पुरेशा असतात. या three specific conditions तपासून दोन त्रिकोण समरूप आहेत का हे ठरविता येते. अशा पुरेशा अटींचा समूह म्हणजेच test for similarity of triangles होत. म्हणून दोन त्रिकोण समरूप आहेत का हे ठरवण्यासाठी त्या विशिष्ट अटी तपासणे पुरेसे असते.

AAA test for similarity of triangles

“For a given correspondence of vertices, when corresponding angles of two triangles are congruent, then the two triangles are similar”.

In ΔABC and ΔPQR , in the correspondence $ABC \leftrightarrow PQR$ if

$\angle A \cong \angle P, \angle B \cong \angle Q$ and $\angle C \cong \angle R$ then $\Delta ABC \sim \Delta PQR$

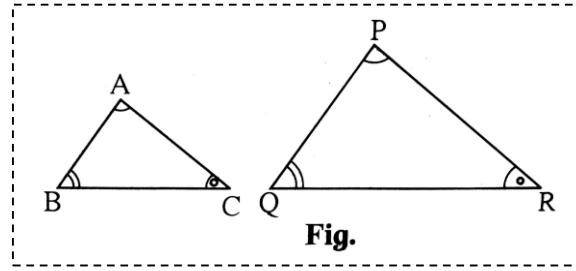


Fig.

Proof of AAA test

Given: In ΔABC and ΔPQR ,
 $\angle A \cong \angle P, \angle B \cong \angle Q$ and $\angle C \cong \angle R$

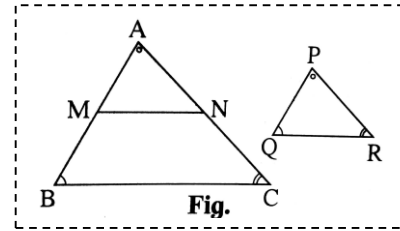


Fig.

To prove: $\Delta ABC \sim \Delta PQR$

Let us assume that ΔABC is bigger than ΔPQR , Mark point M on AB, and point N on AC such that $AM = PQ$ and $AN = PR$.

Show that $\Delta AMN \cong \Delta PQR$, Hence show that $MN \parallel BC$.

Now using basic proportionality theorem, $\frac{AM}{MB} = \frac{AN}{NC}$

That is $\frac{MB}{AM} = \frac{NC}{AN}$ (By invertendo)

$\frac{MB+AM}{AM} = \frac{NC+AN}{AN}$ (By componendo)

$$\therefore \frac{AB}{AM} = \frac{AC}{AN}$$

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR}$$

Similarly it can be shown that $\frac{AB}{PQ} = \frac{BC}{QR}$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

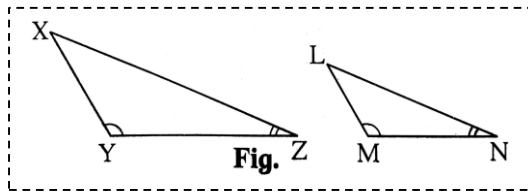
$\therefore \Delta ABC \sim \Delta PQR$

AAA test for similarity of triangles (समरूपतेची कोकोको कसोटी)

दोन triangle च्या vertices मधील given one to one correspondence नुसार होणारे corresponding angle जर congruent (एकरूप) असतील तर ते triangle similar असतात.

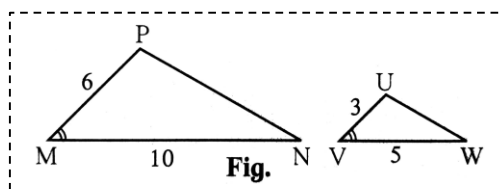
Solved Examples

Ex. (i) In ΔXYZ , $\angle Y = 100^\circ$, $\angle Z = 30^\circ$, In ΔLMN , $\angle M = 100^\circ$, $\angle N = 30^\circ$, Are ΔXYZ and ΔLMN similar? If yes, by which test?



Solution: In ΔXYZ and ΔLMN ,
 $\angle Y = 100^\circ$, $\angle M = 100^\circ$,
 $\therefore \angle Y \cong \angle M$
 $\angle Z = 30^\circ$, $\angle N = 30^\circ$, $\therefore \angle Z \cong \angle N$
 $\Delta XYZ \sim \Delta LMN$ by AA test

Ex. (ii) Are two triangles in figure 1.51 similar, according to the information given? If yes, by which test?



Solution: In ΔPMN and ΔUVW ,

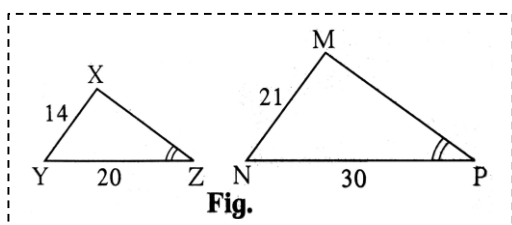
$$\frac{PM}{UV} = \frac{6}{3} = \frac{2}{1}, \frac{MN}{VW} = \frac{10}{5} = \frac{2}{1}$$

$$\therefore \frac{PM}{UV} = \frac{MN}{VW}$$

And $\angle M \cong \angle V$ Given

$\Delta PMN \sim \Delta UVW$ SAS test of similarity

Ex. (iii) Can we say that the two triangles in figure 1.52 similar, according to information given? If yes, by which test?



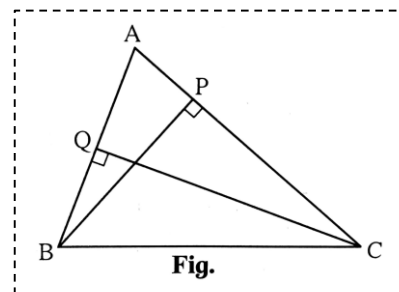
Solution: ΔXYZ and ΔMNP ,

$$\frac{XY}{MN} = \frac{14}{21} = \frac{2}{3}, \frac{YZ}{NP} = \frac{20}{30} = \frac{2}{3}$$

It is given that $\angle Z \cong \angle P$. But $\angle Z$ and $\angle P$ are not included angles by sides which are in proportion.

$\therefore \Delta XYZ$ and ΔMNP cannot be said to be similar.

Ex. (iv) In the adjoining figure $BP \perp AC$, $CQ \perp AB$, $A - P - C$, $A - Q - B$, then prove that ΔAPB and ΔAQC are similar.



Solution: In ΔAPB and ΔAQC

$$\angle APB = 90^\circ \quad \text{..... (I)}$$

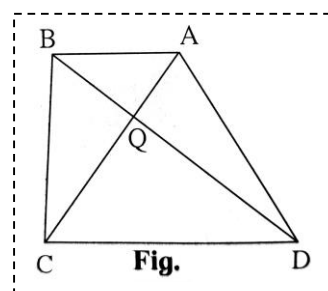
$$\angle AQC = 90^\circ \quad \text{..... (II)}$$

$$\therefore \angle APB \cong \angle AQC \quad \text{....from (I) and (II)}$$

$$\angle PAB \cong \angle QAC \quad \text{(Common angle)}$$

$$\therefore \Delta APB \sim \Delta AQC \quad \text{.... (AA test)}$$

Ex. (v) Diagonals of a quadrilateral ABCD intersect in point Q. If $2QA = QC$, $2QB = QD$, then prove that $DC = 2AB$.



Given: $2QA = QC$, $2QB = QD$

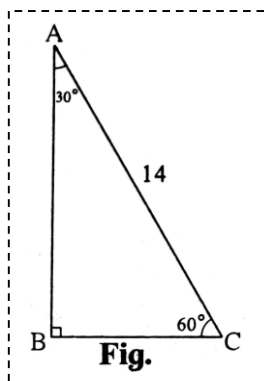
To prove: $CD = 2AB$

$$\text{Proof: } 2QA = QC \quad \therefore \frac{QA}{QC} = \frac{1}{2} \quad \text{..... (I)}$$

Q.5 Figure मध्ये show केल्याप्रमाणे construction नुसार $\square ASRB$ मध्ये त्या properties नुसार आणि $\triangle TAS$, $\triangle TBQ$, $\triangle TAP$ आणि $\triangle TBR$ मध्ये Pythagoras theorem नुसार solve करण्याचा प्रयत्न करावा. मिळणाऱ्या equations नुसार आवश्यक ती identity मिळवावी.

Solved Examples

Ex. (i) See fig, in $\triangle ABC$, $\angle B = 90^\circ$, $\angle A = 30^\circ$, $AC = 14$, then find AB and BC



Solution:

In $\triangle ABC$,

$\angle B = 90^\circ$, $\angle A = 30^\circ$, $\therefore \angle C = 60^\circ$

By 30° - 60° - 90° theorem,

$$BC = \frac{1}{2} \times AC$$

$$BC = \frac{1}{2} \times 14$$

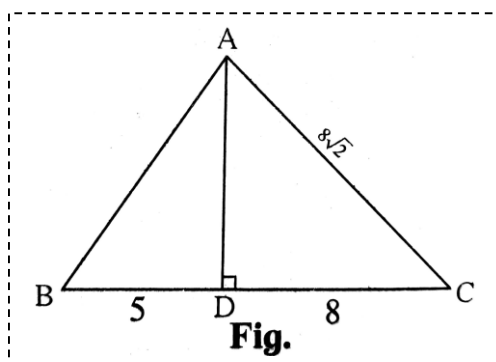
$$BC = 7$$

$$AB = \frac{\sqrt{3}}{2} \times AC$$

$$AB = \frac{\sqrt{3}}{2} \times 14$$

$$AB = 7\sqrt{3}$$

Ex. (ii) See fig, In $\triangle ABC$, seg $AD \perp$ seg BC , $\angle C = 45^\circ$, $BD = 5$ and $AC = 8\sqrt{2}$ then find AD and BC.



Solution: In $\triangle ADC$

$\angle ADC = 90^\circ$, $\angle C = 45^\circ$, $\angle DAC = 45^\circ$

$AD = DC = \frac{1}{\sqrt{2}} \times 8\sqrt{2}$... by 45° - 45° - 90° theorem

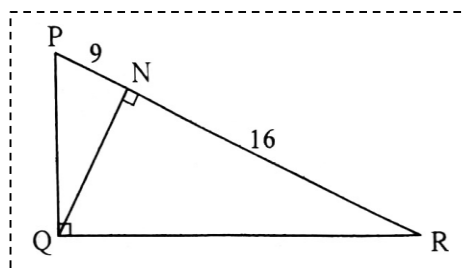
$DC = 8 \therefore AD = 8$

$BC = BD + DC$

$$= 5 + 8$$

$$\therefore BC = 13$$

Ex. (iii) In fig, $\angle PQR = 90^\circ$, seg $QN \perp$ seg PR , $PN = 9$, $NR = 16$. Find QN.



Solution: In $\triangle PQR$, seg $QN \perp$ seg PR

$NQ^2 = PN \times NR$... theorem of geometric mean

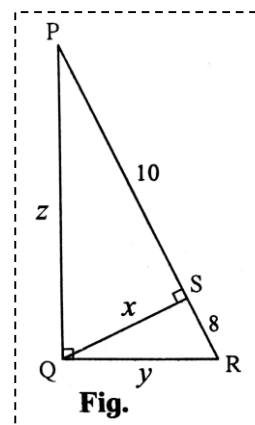
$$\therefore NQ = \sqrt{PN \times NR}$$

$$= \sqrt{9 \times 16}$$

$$= 3 \times 4$$

$$= 12$$

Ex. (iv) See fig, In $\triangle PQR$, $\angle PQR = 90^\circ$, seg $QS \perp$ seg PR then find x, y, z.



Solution: In $\triangle PQR$, $\angle PQR = 90^\circ$, seg $QS \perp$ seg PR

$QS = \sqrt{PS \times SR}$ (theorem of geometric mean)

$$= \sqrt{10 \times 8}$$

$$= \sqrt{5 \times 2 \times 8}$$

$$= \sqrt{5 \times 16}$$

$$= 4\sqrt{5}$$

$$\therefore x = 4\sqrt{5}$$

In ΔQSR , by Pythagoras theorem

$$QR^2 = QS^2 + SR^2$$

$$= (4\sqrt{5})^2 + 8^2$$

$$= 16 \times 5 + 64$$

$$= 80 + 64$$

$$= 144$$

$$\therefore QR = 12$$

In ΔPSQ , by Pythagoras theorem

$$PQ^2 = QS^2 + PS^2$$

$$= (4\sqrt{5})^2 + 10^2$$

$$= 16 \times 5 + 100$$

$$= 80 + 100$$

$$= 180$$

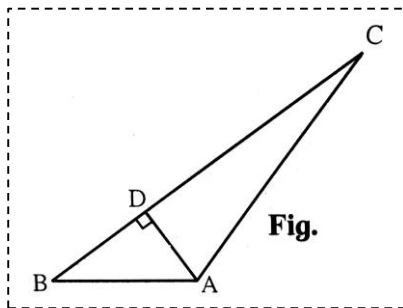
$$= 36 \times 5$$

$$\therefore PQ = 6\sqrt{5}$$

$$\text{Hence } x = 4\sqrt{5}, y = 12, z = 6\sqrt{5}$$

Example (i) ते (iv) साठी- Property of $30^\circ - 60^\circ - 90^\circ$, $45^\circ - 45^\circ - 90^\circ$, theorem of geometric mean आणि pythagorean theorem चा use करून example मध्ये दिलेल्या information नुसार solve करण्याचा प्रयत्न करावा.

Ex. (v) See fig 2.16, In ΔABC , seg $AD \perp$ seg BC , Prove that: $AB^2 + CD^2 = BD^2 + AC^2$



Solution: According to Pythagoras theorem, in ΔADC

$$AC^2 = AD^2 + CD^2$$

$$\therefore AD^2 = AC^2 - CD^2 \quad \dots (I)$$

In ΔADB

$$AB^2 = AD^2 + BD^2$$

$$\therefore AD^2 = AB^2 - BD^2 \quad \dots (II)$$

$$\therefore AB^2 - BD^2 = AC^2 - CD^2 \quad \dots \text{[From I and II]}$$

$$\therefore AB^2 + CD^2 = AC^2 + BD^2$$

Example (v) - ΔADC आणि ΔADB मध्ये pythagorean नुसार equation (i) आणि (ii) मिळतात. त्यावरून prove करण्यासाठी सांगितलेली expression मिळवावी.

Problem Set - 2

Q.1 Some questions and their alternative answers are given. Select the correct alternative:

(i) Out of the following which is the Pythagorean triplet?

- | | |
|----------------|---------------|
| (A) (1, 5, 10) | (B) (3, 4, 5) |
| (C) (2, 2, 2) | (D) (5, 5, 2) |

Ans: (B)

(ii) In a right angled triangle, if the sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

- (A) 15 (B) 13 (C) 5 (D) 12

Ans: (B)

(iii) Out of the dates given below which date constitutes a Pythagorean triplet?

- | | |
|--------------|--------------|
| (A) 15/08/17 | (B) 16/08/16 |
| (C) 3/5/17 | (D) 4/9/15 |

Ans: (A)

(iv) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.

- (A) Obtuse angled triangle
(B) Acute angled triangle
(C) Right angled triangle
(D) Equilateral triangle

Ans: (C)

(v) Find perimeter of a square if its diagonal is $10\sqrt{2}$ cm.

- (A) 10 cm (B) $40\sqrt{2}$ cm (C) 20 cm (D) 40 cm

Ans: (D)

(vi) Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

- (A) 9 cm (B) 4 cm (C) 6 cm (D) $2\sqrt{6}$ cm

Ans: (C)

(vii) Height and base of a right angled triangle are 24 cm and 18 cm; find the length of its hypotenuse.

- (A) 24 cm (B) 30 cm (C) 15 cm (D) 18 cm

Ans: (B)

(viii) In ΔABC , $AB = 6\sqrt{3}$ cm, $AC = 12$ cm, $BC = 6$ cm. Find measure of $\angle A$.

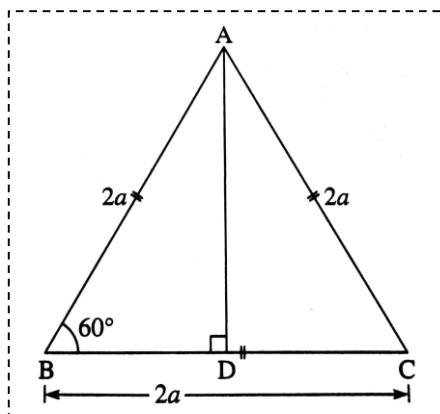
- (A) 30° (B) 60° (C) 90° (D) 45°

Ans: (A)

Q.2 Solve the following examples:

(i) Find the height of an equilateral triangle having side $2a$.

Solution:



Suppose the given equilateral triangle be ABC and seg AD be the height

$\angle B = 60^\circ \dots$ (Angle of an equilateral triangle) ... (1)

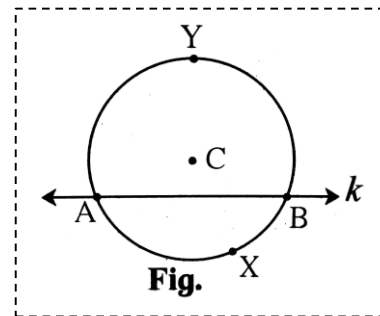
In ΔADB ,

$$\angle ABD + \angle BAD + \angle ADB = 180^\circ$$

3.6 Arc of a circle

3.6.1 Arc of the circle:

- A secant divides a circle in two parts. Any one of these two parts and the common points of the circle and the secant constitute an **arc of the circle**.
- The points of intersection of circle and secant are called the end points of the arcs.



- In this figure, due to secant k we get two arcs of the circle with centre C -arc AYB , arc AXB .

iv) Major arc: If the centre of a circle is on one side of the secant then the arc on the side of the centre is called 'major arc'.

v) Minor arc: The arc which is on the other side of the centre is called 'minor arc'.

In the figure arc AYB is a major arc and arc AXB is a minor arc.

vi) If there is no confusion then the name of a minor arc is written using its end points only. For example, the arc AXB in figure, is written as arc AB .

vii) Here after, we are going to use the same convention for writing the names of arcs.

Arc of a circle (वर्तुळकंस) -

Secant (वृत्तछेदिका) मुळे circle चे दोन भागांत विभाजन होते. यांपैकी कोणताही एक part आणि secant चे circle वरील point मिळून होणाऱ्या figure ला Arc of a circle म्हणतात.

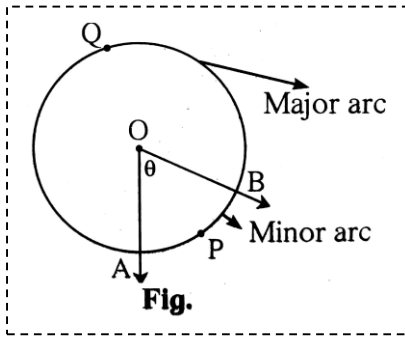
Secant च्या ज्या बाजूला centre of circle (वर्तुळकेंद्र) असते त्या side च्या arc ला major arc आणि opposite side (विरुद्ध बाजू) च्या arc (कंस) ला minor arc म्हणतात. वरील figure मध्ये arc AYB हा major arc (विशालकंस) आणि arc AXB हा minor arc (लघुकंस) आहे. एखाद्या arc of circle चे नाव three letters

वापरून लिहिल्याने तो नेमका समजतो, परंतु काही confusion (संदिग्धता) निर्माण होत नसेल तर minor arc चे नाव त्याचे end point (अंत्यबिंदू) दर्शवणाऱ्या two letters नी लिहितात. उदा. वरील figure मधील कंस AXB हा कंस AB असाही लिहितात.

3.6.2 Central angle

When the vertex of an angle is the centre of a circle, it is called a central angle. In the figure, O is the centre of a circle and $\angle AOB$ is a central angle.

Like secant, a central angle also divides a circle into two arcs.



Central angle (केंद्रीय कोन) -

The vertex of an angle हा centre of a circle वर असतो. त्या angle ला Central angle म्हणतात.

Figure मध्ये centre O असेलेले circle असून $\angle AOB$ हा Central angle आहे.

Secant प्रमाणे Central angle मुळे सुद्धा circle चे दोन arc मध्ये विभाजन होते.

3.6.3 Measure of an arc

To compare two arcs, we need to know their measures. Measure of an arc is defined as follows.

(i) Measure of a minor arc is equal to the measure of its corresponding central angle. In figure, measure of central $\angle AOB$ is θ .

$\therefore$ measure of minor arc APB is also θ .

(ii) Measure of major arc

= 360° - measure of corresponding minor arc.

In figure, measure of major arc AQB

= 360° - measure of minor arc APB = $360^\circ - \theta$

(iii) Measure of a semicircular arc, that is of a semi-circle, is 180° .

(iv) Measure of a complete circle is 360° .

एकमेकांना एकाच point मध्ये intersect करतात. अशा circles ना touching circles (स्पर्शवर्तुळे) म्हणतात.

Touching circles ची definition पुढीलप्रमाणे करता येते.

एका plane मधील दोन circles त्याच plane मधील एका line ला same point मध्ये छेदत असतील, तर त्यांना touching circles म्हणतात. ती line दोन्ही circle ची common tangent (सामाईक स्पर्शिका) असते.

दोन्ही circles व line यांच्यातील common point ला common point of contact (सामाईक स्पर्शबिंदू) म्हणतात.

Figure (A) मध्ये centre R व S असणारी circles line l ला T या एकाच बिंदूत छेदतात. म्हणून ती दोन्ही touching circles असून line l ही त्यांची common tangent (सामाईक स्पर्शिका) आहे. Figure मधील circles ही externally touching (बाह्यस्पर्शी) आहेत.

Figure (B) मधील circles ही internally touching (अंतर्स्पर्शी) असून line p ही त्यांची tangent आहे.

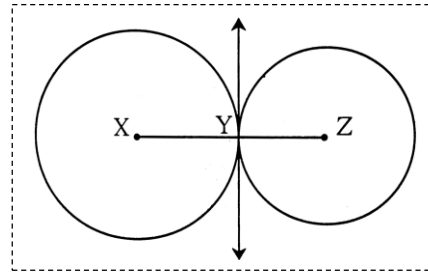
Activity I:

Take three collinear points X-Y-Z as shown in figure,

i) Draw a circle with centre X and radius XY.

ii) Draw another circle with centre Z and radius YZ.

Note that both the circles intersect each other at the single point Y.



iii) Draw a line through point Y and perpendicular to seg XZ.

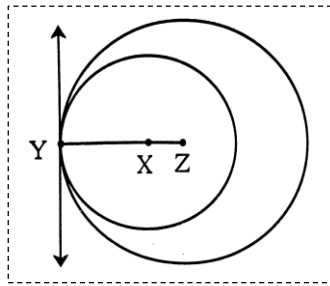
Note that this line is a common tangent of the two circles.

Activity II:

Take points Y-X-Z as shown in the figure.

Draw a circle with centre Z and radius ZY.

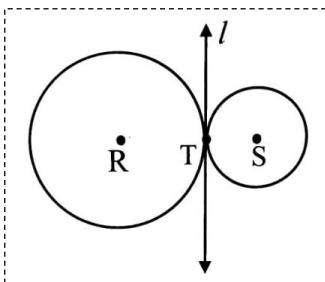
Also draw a circle with centre X and radius XY.



Note that both the circles intersect each other at the point Y.

Draw a line perpendicular to seg YZ through point Y, which is the common tangent for the circles.

Let's think

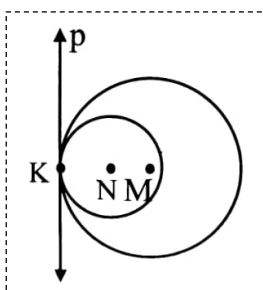


1) The circles shown in the figure are called externally touching circles. Why?

Ans: Circles with centres R and S lie in the same plane and intersect with a line l in the plane in one and only one point T [R-T-S].

Hence, the given circles are externally touching circles.

2) The circles shown in the figure are called internally touching circles. Why?



4.2 Construction of similar Triangle:

i) To construct a triangle similar to the given triangle, satisfying the condition of given ratio of corresponding sides.

ii) The corresponding sides of similar triangles are in the same proportion and the corresponding angles of these triangles are equal. Using this property, a triangle which is similar to the given Triangle can be constructed.

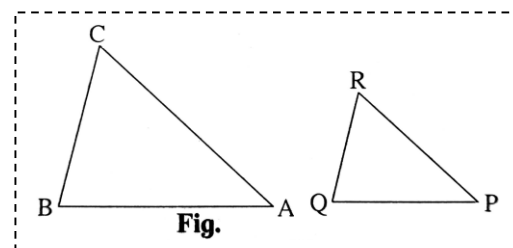
Construction of similar Triangle (समरूप त्रिकोणाची रचना) -

एका triangle च्या side दिल्या असता, त्याच्याशी simillel असणारा आणि ratio ची condition (अट) पूर्ण करणारा triangle काढणे.

दोन similar triangles च्या corresponding sides (संगत बाजू) same proportion (एकाच प्रमाणात) मध्ये असतात आणि त्यांचे corresponding angles (संगत कोन) equal (एकरूप) असतात. याचा उपयोग करून दिलेल्या triangle शी similar असणारा त्रिकोण काढता येतो.

Ex. (i) $\Delta ABC \sim \Delta PQR$, in ΔABC , $AB = 5.4$ cm, $BC = 4.2$ cm, $AC = 6.0$ cm. $AB:PQ = 3:2$. Construct ΔABC and ΔPQR .

Rough figure:



Construct ΔABC of given measure.

ΔABC and ΔPQR are similar.

$\therefore$ their corresponding sides are proportional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2} \dots\dots\dots (I)$$

As the sides AB, BC, AC are known, we can find the lengths of sides PQ, QR, PR.

$$\text{Using equation (I), } \frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6.0}{PR} = \frac{3}{2}$$

$$\frac{5.4}{PQ} = \frac{3}{2} \therefore 3 \times PQ = 5.4 \times 2 \therefore PQ = \frac{10.8}{3}$$

$$\therefore PQ = 3.6 \text{ cm}$$

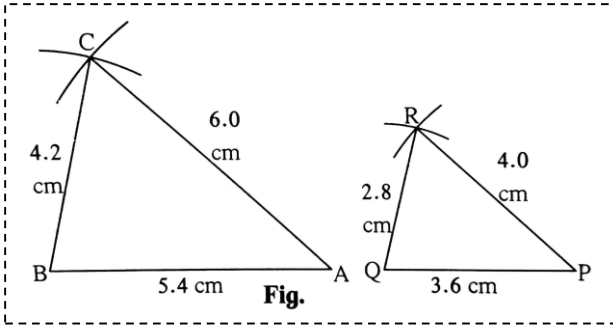
$$\frac{4.2}{QR} = \frac{3}{2} \therefore 3 \times QR = 4.2 \times 2 \therefore QR = \frac{8.4}{3}$$

$$\therefore QR = 2.8 \text{ cm}$$

$$\frac{6.0}{PR} = \frac{3}{2} \therefore 3 \times PR = 6.0 \times 2 \therefore PR = \frac{12.0}{3}$$

$$\therefore PR = 4.0 \text{ cm}$$

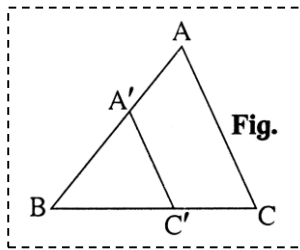
$$\therefore PQ = 3.6 \text{ cm}, QR = 2.8 \text{ cm} \text{ and } PR = 4.0 \text{ cm}.$$



Example (i) ΔABC व ΔPQR मधील similarity नुसार ΔPQR च्या सर्व sides च्या length find out कराव्यात. Triangle च्या three sides दिल्य असता triangle draw करण्याची Steps follow करण्यात आणि त्यानुसार similar triangle draw करावेत.

Ex. (ii) Construct any ΔABC . Construct $\Delta A'BC'$ such that $AB:A'B=5:3$ and $\Delta ABC \sim \Delta A'BC'$.

Analysis:



i) As shown in figure, let the points B, A, A' and B, C, C' are collinear.

$$\Delta ABC \sim \Delta A'BC' \therefore \angle ABC = \angle A'BC'$$

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{5}{3}$$

ii) Sides of ΔABC are longer than corresponding sides of $\Delta A'BC'$.

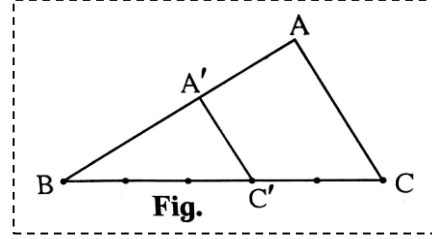
$\therefore$ the length of side BC' will be equal to 3 parts out of 5 equal parts of side BC.

iii) So if we construct ΔABC , point C' will be on the side BC, at a distance equal to 3 parts from B.

iv) Now A' is the point of intersection of AB and a line through C', which is parallel to CA.

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{3}{5} \text{ i.e. } \frac{BA}{BA'} = \frac{BC}{BC'} = \frac{5}{3} \dots\dots\dots \text{taking inverse}$$

Steps of construction:



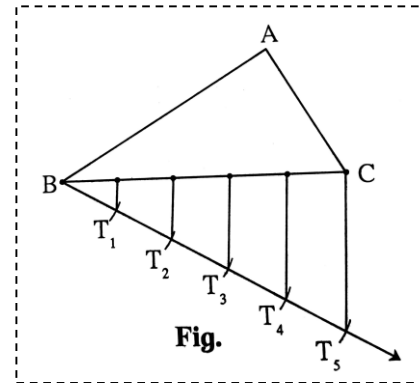
i) Construct a ΔABC .

ii) Divide segment BC in 5 equal parts.

iii) Name the endpoints of third part of segment BC as C'. $\therefore BC' = \frac{3}{5} BC$.

iv) Now draw a line parallel to AC through C'. Name the point where the parallel line intersects AB as A'.

v) $\Delta A'BC'$ is the required triangle similar to ΔABC .



Steps of construction (रचनेच्या पायऱ्या):

1) ΔABC हा कोणताही एक triangle काढा.

2) Line segment BC चे पाच equal part (समान भाग) करा.

3) बिंदू B पुढील third point ला C' नाव द्या.

$$\therefore BC' = \frac{3}{5} BC$$

4) आता C' मधून line segment CA ला parallel line काढा. ती line segment AB ला जेथे छेदते, त्या point ला A' नाव द्या.

5) ΔABC शी similar (समरूप) असणारा $\Delta A'BC'$ हा required triangle (इष्ट त्रिकोण) आहे.

Practice set 4.1

Q. 1 $\triangle ABC \sim \triangle LMN$. In $\triangle ABC$, $AB = 5.5$ cm, $BC = 6$ cm, $CA = 4.5$ cm. Construct $\triangle ABC$ and $\triangle LMN$ such that $\frac{BC}{MN} = \frac{5}{4}$.

Analysis:

i) For $\triangle ABC$, the lengths of three sides are known.

$\therefore \triangle ABC$ can be constructed.

ii) $\triangle ABC \sim \triangle LMN$

$\therefore \frac{AB}{LM} = \frac{BC}{MN} = \frac{AC}{LN}$ (Corresponding sides of similar triangles are in the proportion)

iii) $\therefore \frac{5.5}{LM} = \frac{6}{MN} = \frac{4.5}{LN} = \frac{5}{4}$

$\therefore \frac{5.5}{LM} = \frac{5}{4} \therefore LM = \frac{5.5 \times 4}{5} \therefore LM = 1.1 \times 4 \therefore LM = 4.4$ cm

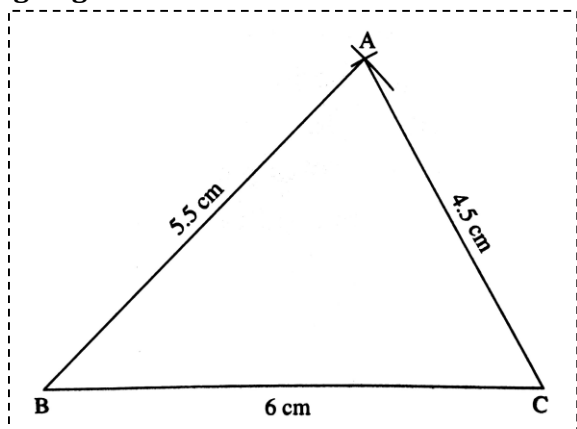
$\therefore \frac{6}{MN} = \frac{5}{4} \therefore MN = \frac{6 \times 4}{5} \therefore MN = \frac{24}{5} \therefore MN = 4.8$ cm

$\therefore \frac{4.5}{LN} = \frac{5}{4} \therefore LN = \frac{4.5 \times 4}{5} \therefore LN = \frac{18}{5} \therefore LN = 3.6$ cm

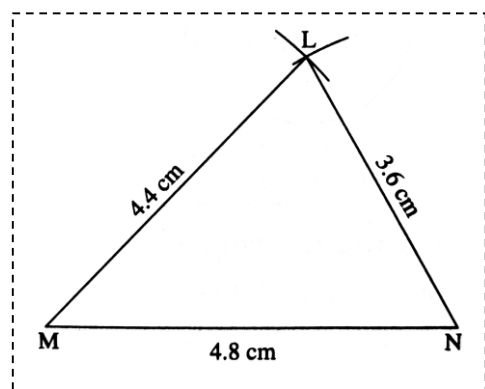
iv) For $\triangle LMN$, the lengths of three sides are known.

$\therefore \triangle LMN$ can be constructed.

Rough figure:



Construction:



Q. 2 $\triangle PQR \sim \triangle LTR$. In $\triangle PQR$, $PQ = 4.2$ cm, $QR = 5.4$ cm, $PR = 4.8$ cm. Construct $\triangle PQR$ and $\triangle LTR$ such that $\frac{PQ}{LT} = \frac{3}{4}$.

Analysis:

i) As shown in the figure, Let R-P-L and R-Q-T.

ii) $\triangle PQR \sim \triangle LTR$ (Given)

$\therefore \angle PRQ \cong \angle LRT$

.... (Corresponding angles of similar triangles)

$\frac{PQ}{LT} = \frac{QR}{TR} = \frac{PR}{LR}$ (i)

... (Corresponding sides of similar triangles)

But, $\frac{PQ}{LT} = \frac{3}{4}$ (ii) (Give)

$\therefore \frac{PQ}{LT} = \frac{QR}{TR} = \frac{PR}{LR} = \frac{3}{4}$ [From (i) and (ii)]

$\therefore$ Sides of $\triangle LTR$ are longer than corresponding sides of $\triangle PQR$.

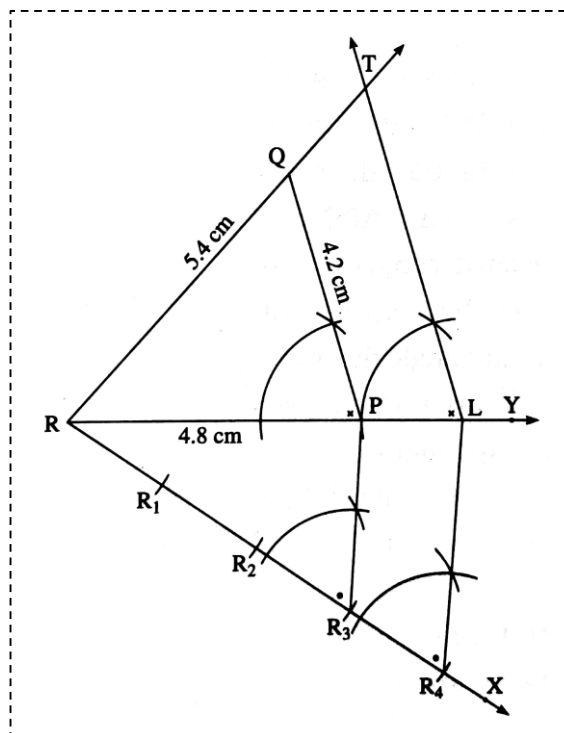
$\therefore$ If seg QR is divided into 3 equal parts, then seg TR will be 4 times each part of seg QR.

iii) So, if we construct $\triangle PQR$, point T will be on side RQ, at a distance equal to 4 parts from R.

iv) Now point L is the point of intersection of ray RP and a line through T, parallel to PQ.

$\therefore \triangle LTR$ is the required triangle similar to $\triangle PQR$.

Construction:



To construct tangents to a circle from a point outside the circle (दिलेल्या वर्तुळाला त्याबाहेरील दिलेल्या बिंदूतून स्पर्शिका काढणे).

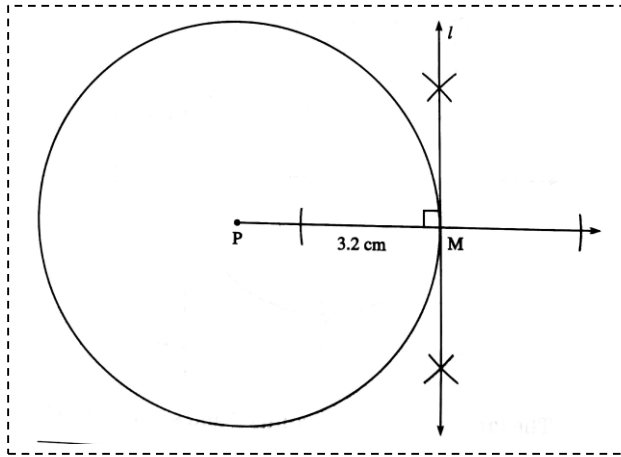
Steps of construction (रचनेच्या पायऱ्या):

- 1) Centre O असलेले कोणत्याही radius चे एक circle काढा.
- 2) circle च्या exterior (बाह्यभाग) मध्ये P हा एक point घ्या.
- 3) Line segment OP काढा. रेख OP चा perpendicular bisector (लंबदुभाजक) काढून midpoint (मध्यबिंदू) M मिळवा.
- 4) केंद्र M व radius OM घेऊन arc of circle काढा.
- 5) हा arc of circle (वर्तुळकंस) दिलेल्या circle ला A आणि B point मध्ये छेदतो.
- 6) line PA व line PB काढा. Line PA व line PB ह्या वर्तुळाच्या required tangent (अपेक्षित स्पर्शिका) आहेत.

Practice set 4.2

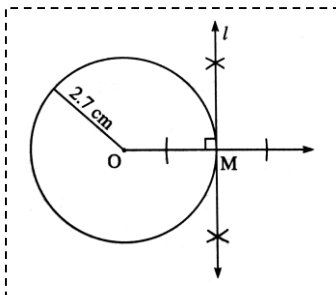
Q. 1 Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it.

Ans:



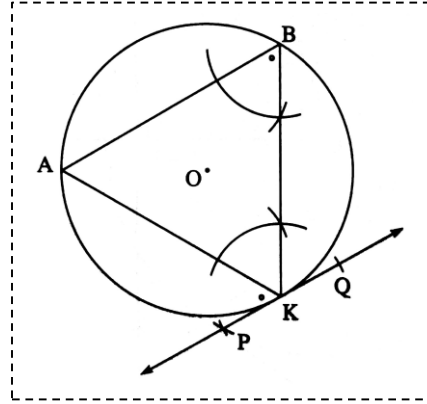
Q. 2 Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it.

Ans:



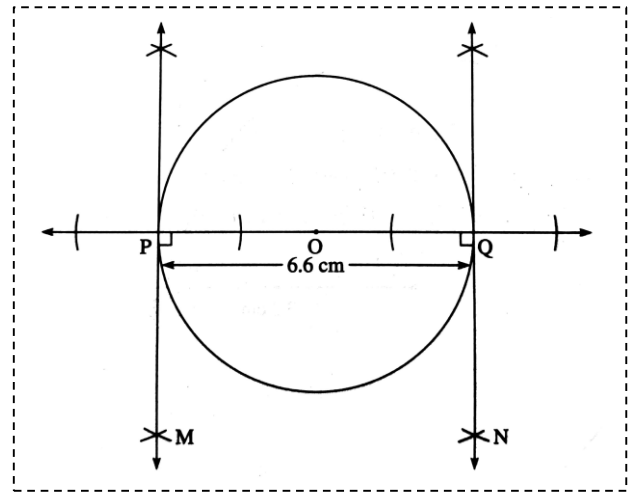
Q. 3 Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre.

Ans:



Q. 4 Draw a circle of radius 3.3 cm. Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your observation about the tangents.

Ans:



The tangents drawn at points P and Q are parallel.

Q. 5 Draw a circle of radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. Construct tangents at point M and N to the circle.

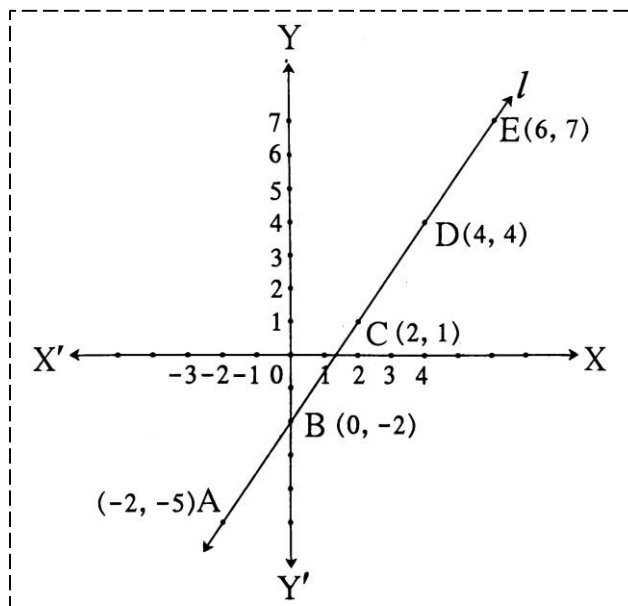
Ans:

5.7 Slope of a line

When we walk on a plane road we need not exert much effort but while climbing up a slope we need more effort. In science, we have studied that while climbing up a slope we have to work against gravitational force. In co-ordinate geometry, slope of a line is an important concept. We will learn it through the following activity.

Activity I

In the figure points A(-2, -5), B(0,-2), C(2,1), D(4,4), E(6,7) lie on line l. Observe the table which is made with the help of coordinates of these points on line l.



3	D	A	(4, 4)	(-2, -5)	$\frac{-5-4}{-2-4} = \frac{-9}{-6} = \frac{3}{2}$
4	B	C	(0, -2)	(2, 1)	$\frac{1-(-2)}{2-0} = \frac{3}{2}$
5	C	A	(2, 1)	(-2, -5)	$\frac{-5-1}{-2-2} = \frac{-6}{-4} = \frac{3}{2}$
6	A	C	(-2, -5)	(2, 1)	$\frac{1-(-5)}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$

Fill in the blank spaces in the above table. Similarly take some other pairs of points on line l and ratio $\frac{y_2 - y_1}{x_2 - x_1}$ for each pair.

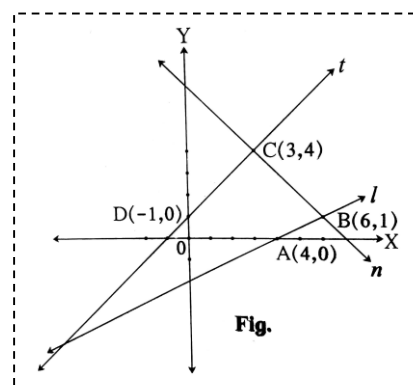
Form this activity, we understand that for any two points (x_1, y_1) and (x_2, y_2) on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is constant.

If (x_1, y_1) and (x_2, y_2) are any two points on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is called the slope of the line l.

Generally slope is shown by letter $\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$

Activity II:

In the figure, some points on line l, t and n are given. Find the slopes of those lines.



Now you will know,

Sr. No.	First point	Second point	Co-ordinates of the first point (x_1, y_1)	Co-ordinates of the second point (x_2, y_2)	$\frac{y_2 - y_1}{x_2 - x_1}$
1	C	E	(2, 1)	(6, 7)	$\frac{7-1}{6-2} = \frac{6}{4} = \frac{3}{2}$
2	A	D	(-2, -5)	(4, 4)	$\frac{4-(-5)}{4-(-2)} = \frac{9}{6} = \frac{3}{2}$

- (i) Slopes of line l and line t are positive.
- (ii) Slope of line n is negative.
- (iii) Slope of line t is more than slope of line l .
- (iv) Slopes of lines l and t which make acute angles with X- axis are positive.
- (v) Slope of line n making obtuse angle with X- axis is negative.

या कृतीतून लक्षात येते की, line l च्या (x_1, y_1) आणि (x_2, y_2) या कोणत्याही दोन point साठी $\frac{y_2 - y_1}{x_2 - x_1}$ हा ratio constant (स्थिर) असतो.

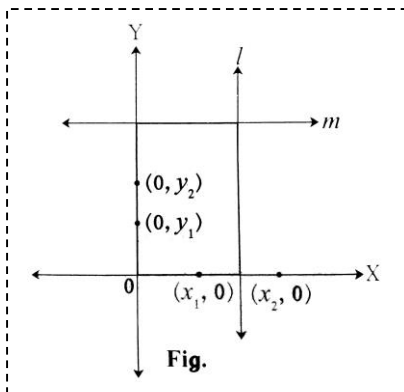
line l चे (x_1, y_1) आणि (x_2, y_2) हे कोणतेही दोन point असतील, तर $\frac{y_2 - y_1}{x_2 - x_1}$ ह्या स्थिर ratio ला line l चा slope (चढ) म्हणतात.

Slope of line (रेषेचा चढ) सामान्यपणे m या अक्षराने दाखवतात.

$$\therefore m = \frac{y_2 - y_1}{x_2 - x_1}$$

5.7.1 Slopes of X-axis, Y-axis and lines parallel to axes

In the figure, $(x_1, 0)$ and $(x_2, 0)$ are two points on the X-axis.



$$\text{Slope of X-axis} = \frac{0-0}{x_2-x_1} = 0$$

In the same way $(0, y_1)$ and $(0, y_2)$ are two points on the Y- axis.

$$\text{Slope of Y-axis} = \frac{y_2 - y_1}{0 - 0} = \frac{y_2 - y_1}{0}$$

But division by 0 is not possible.

$\therefore$ slope of Y- axis cannot be determined.

Now try to find the slope of any line like line m which is parallel to X- axis. It will come out to be 0.

Similarly we cannot determine the slope of a line like line l which is parallel to Y- axis.

5.7.2 Slope of line — using ratio in trigonometry

In the figure, points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are on line l . Line l intersects X axis in point T .

seg $QS \perp$ X- axis, seg $PR \perp$ seg QS

$\therefore$ seg $PR \parallel$ seg TS corresponding angle test

$\therefore QR = y_2 - y_1$ and $PR = x_2 - x_1$

$$\therefore \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{..... (I)}$$

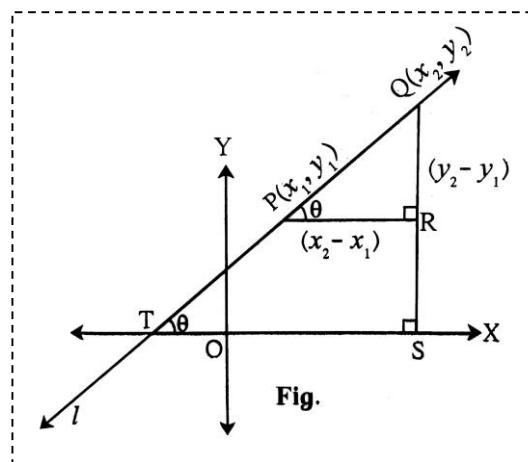
Line TQ makes an angle θ with the X- axis

$$\therefore \frac{QR}{PR} = \tan \theta \quad \text{..... (II)}$$

$$\text{From (I) and (II), } \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta \quad \therefore m = \tan \theta$$

Now seg $PR \parallel$ seg TS , line l is transversal

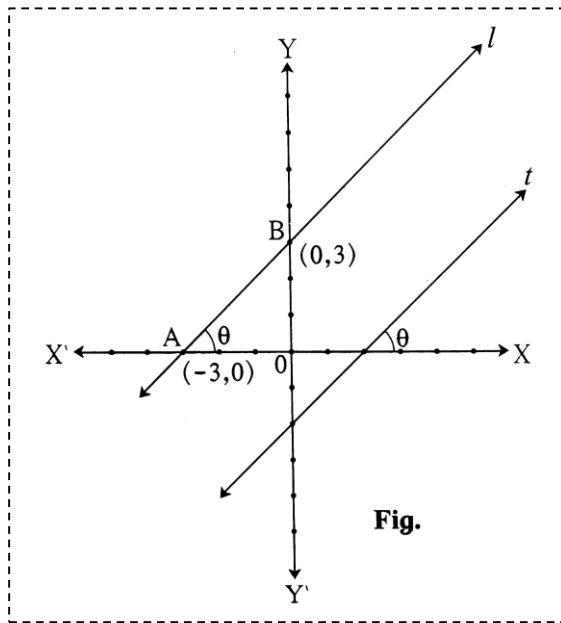
$\therefore \angle QPR = \angle QTS$ [Corresponding angles]



From this, we can define slope in another way. The tan ratio of an angle made by the line with the positive direction of X-axis is called the slope of that line. When any two lines have same slope, these lines make equal angles with the positive direction of X- axis. These two lines are parallel.

5.7.3 Slope of Parallel Lines

Activity:



In the figure both line l and line t make angle θ with the positive direction of X -axis.

$\therefore$ line $l \parallel$ line t [Corresponding angle test]

Consider, point $A(-3, 0)$ and point $B(0, 3)$ on line l

Find the slope of line l .

$$\text{Slope of line } l = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{0 - (-3)} = \frac{3}{3} = 1$$

In the similar way, consider suitable points on the line t and find the slope of line t .

From this, you can verify that parallel lines have equal slopes.

Here $\theta = 45^\circ$.

Use slope, $m = \tan \theta$ and verify that slopes of parallel lines are equal.

Similarly taking $\theta = 30^\circ$, $\theta = 60^\circ$ verify that slopes of parallel lines are equal.

Slope of Parallel Lines (समांतर रेषांचा चढ) -

Figure मध्ये line l आणि line t या दोन्ही रेषांनी X -axis च्या positive direction (धन दिशा) शी केलेला angle θ आहे.

या ठिकाणी $\theta = 45^\circ$ आहे.

Slope (चढ), $m = \tan \theta$ हे वापरूनही दोन्ही parallel lines चे slope equal येतात हे पडताळून पाहा.

याप्रमाणेच $\theta = 30^\circ$, $\theta = 60^\circ$ घेऊन parallel lines (समांतर रेषा) चे slope equal असतात याचा पडताळा घ्या.

Remember this!

The slope of X -axis and of any line parallel to X -axis is zero.

The slope of Y -axis and of any line parallel to Y -axis cannot be determined.

Remember this (हे लक्षात ठेवूया).

X -axis चा किंवा X -axis ला parallel line चा slope (चढ) zero असतो.

Y -axis चा किंवा Y -axis ला parallel line चा slope ठरविता येत नाही.

Solved Examples

Ex. (i) Find the slope of the line passing through the points $A(-3, 5)$, and $B(4, -1)$

Solution: Let, $x_1 = -3$, $x_2 = 4$, $y_1 = 5$, $y_2 = -1$

$$\therefore \text{Slope of line } AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{4 - (-3)} = \frac{-6}{7}$$

Ex. (ii) Show that points $P(-2, 3)$, $Q(1, 2)$, $R(4, 1)$ are collinear.

Solution: $P(-2, 3)$, $Q(1, 2)$ and $R(4, 1)$ are given points

$$\text{Slope of line } PQ = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{1 - (-2)} = -\frac{1}{3}$$

$$\text{Slope of line } QR = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{4 - 1} = -\frac{1}{3}$$

Slope of line PQ and line QR is equal.

But point Q lies on both the lines.

$\therefore$ Point P , Q , R are collinear.

Ex. (iii) If slope of the line joining points $P(k, 0)$ and $Q(-3, -2)$ is $\frac{2}{7}$ then find k .

Solution: $P(k, 0)$ and $Q(-3, -2)$

$$\text{Slope of line } PQ = \frac{-2 - 0}{-3 - k} = \frac{-2}{-3 - k}$$

PROBLEM SET - 6

Q.1 Choose the correct alternative answer for each of the following questions:

(i) $\sin\theta \operatorname{cosec}\theta = ?$ **(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$** **Ans:** (A) 1**(ii) $\operatorname{cosec} 45^\circ = ?$** **(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$** **Ans:** (B) $\sqrt{2}$ **(iii) $1 + \tan^2\theta = ?$ (March-19)****(A) $\cot^2\theta$ (B) $\operatorname{cosec}^2\theta$
(C) $\sec^2\theta$ (D) $\tan^2\theta$** **Ans:** (C) $\sec^2\theta$ **(iv) When we see at a higher level, from the horizontal line, angle formed is****(A) angle of elevation (B) angle of depression
(C) 0 (D) straight line****Ans:** (A) angle of elevation**Q.2 If $\sin\theta = \frac{11}{61}$, find the value of $\cos\theta$ using trigonometric identity.****Sol:** Given that, $\sin\theta = \frac{11}{61}$ (Given)

$$\sin^2\theta + \cos^2\theta = 1 \text{ (Identity)}$$

$$\therefore \left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$$

$$\therefore \cos^2\theta = 1 - \left(\frac{11}{61}\right)^2$$

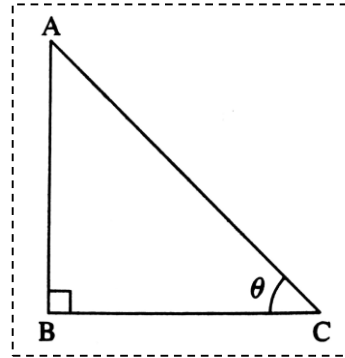
$$\therefore \cos^2\theta = 1 - \frac{121}{3721}$$

$$\therefore \cos^2\theta = \frac{3721-121}{3721}$$

$$\therefore \cos^2\theta = \frac{3600}{3721}$$

Taking square root of both sides, $\therefore \cos\theta = \frac{60}{61}$

Ans: $\cos\theta = \frac{60}{61}$.

Q.3 If $\tan\theta = 2$, find the values of other trigonometric ratios.**Sol:**Consider ΔABC ,Where $\angle ABC = 90^\circ$ and $\angle ACB = \theta$

$$\tan\theta = \frac{AB}{BC}, \tan\theta = 2 \therefore \frac{AB}{BC} = \frac{2}{1}$$

$$\therefore AB = 2k \text{ and } BC = k$$

In right angled ΔABC , by Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\therefore AC^2 = (2k)^2 + (k)^2$$

$$\therefore AC^2 = 4k^2 + k^2$$

$$\therefore AC^2 = 5k^2$$

$$\therefore AC = \sqrt{5}k \text{ (Taking square root of both the sides,)}$$

$$\sin\theta = \frac{AB}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$$

$$\cos\theta = \frac{BC}{AC} = \frac{k}{\sqrt{5}k} = \frac{1}{\sqrt{5}}$$

$$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$$

$$\sec\theta = \frac{AC}{BC} = \frac{\sqrt{5}k}{k} = \frac{\sqrt{5}}{1}$$

Let's think.

Example: As shown in the adjacent figure, a sphere is placed in a cylinder. It touches the top, bottom and the curved surface of the cylinder. If radius of the base of the cylinder is 'r'

(i) What is the ratio of the radii of the sphere and the cylinder?

(ii) What is the ratio of the curved surface area of the cylinder and the surface area of the sphere?

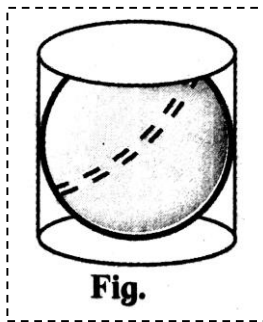
(iii) What is the ratio of the volumes of the cylinder and the sphere?

Sol: Radius of base of cylinder = radius of sphere

∴ Radius of sphere = r

Also, height of cylinder = diameter of sphere

∴ h = d ∴ h = 2r (I)



$$i) \frac{\text{Radius of sphere}}{\text{Radius of cylinder}} = \frac{r}{r} = \frac{1}{1} = 1$$

∴ Radius of sphere : radius of cylinder = 1:1.

$$ii) \frac{\text{Curved surface area of cylinder}}{\text{Surface area of sphere}} = \frac{2\pi rh}{4\pi r^2}$$

$$= \frac{h}{2r} = \frac{2r}{2r} \dots [\text{From (I)}] = \frac{2r}{2r} = \frac{1}{1}$$

∴ Curved surface area of cylinder: surface area of sphere = 1:1.

$$iii) \frac{\text{Volume of cylinder}}{\text{Volume of sphere}} = \frac{\pi r^2 h}{\frac{4}{3}\pi r^3}$$

$$= \frac{3h}{4r} = \frac{3(2r)}{4r} \dots [\text{From (I)}]$$

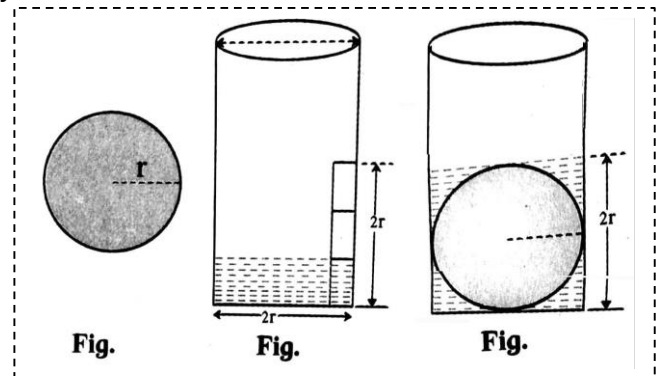
$$\frac{6r}{4r} = \frac{3}{2}$$

∴ Volume of cylinder: volume of sphere = 3:2.

Activity:

Finding volume of a sphere using cylindrical beaker and water

- As shown in the above figures, take a ball and a beaker of the same radius as that of the ball.
- Cut a strip of paper of length equal to the diameter of the beaker. Draw two lines on the strip dividing it into three equal parts. Stick it on the beaker straight up from the bottom.
- Fill water in the beaker up to the first mark of the strip from the bottom. Push the ball in the beaker slowly so that it touches its bottom.
- Observe how much the water level rises.
- You will notice that the water level has risen exactly up to the total height of the strip.
- Try to understand how we get the formula for the volume of the sphere. The shape of the beaker is cylindrical.



Sol: The volume of the part of the beaker up to height 2r can be obtained by the formula of volume of a cylinder.

Let us assume that the volume is V.

$$\therefore V = \pi r^2 h = \pi \times r^2 \times 2r = 2\pi r^3 \quad (\because h = 2r)$$

But V = volume of the ball + volume of the water which was already in the beaker.

$$= \text{volume of the ball} + \frac{1}{3} \times 2\pi r^3$$

$$\therefore \text{Volume of the ball} = V - \frac{1}{3} \times 2\pi r^3$$

$$= 2\pi r^3 - \frac{2}{3}\pi r^3 = \frac{6\pi r^3 - 2\pi r^3}{3} = \frac{4\pi r^3}{3}$$

Hence we get the formula of the volume of a sphere as $V = \frac{4}{3}\pi r^3$

वरील figure मध्ये दाखवल्याप्रमाणे एक ball आणि ball च्या radius एवढीच (r) radius असलेले एक beaker (चंचुपात्र)

घ्या. Beaker च्या diameter एवढ्या (2r) length ची एक strip of paper घ्या. तिच्या length चे three equal parts करणाऱ्या दोन line पट्टीवर काढा. ती strip beaker ला त्याच्या bottom पासून उभी चिकटवा. Beaker मध्ये कागदी पट्टीच्या खालून पहिल्या भागापर्यंत water fill करा नंतर ball beaker मध्ये bottom ला टेकेपर्यंत सावकाश बसवा. Beaker मधील water level कुठपर्यंत rises (वाढली) आहे, हे पाहा.

Water level कागदी पट्टीच्या पूर्ण height पर्यंत आलेली दिसेल. या observation वरून ball च्या volume चा formula (सूत्र) कसे मिळते हे समजून घ्या.

Beaker हा cylindrical (वृत्तचिती) आकाराचे आहे. म्हणून beaker च्या 2r एवढ्या height पर्यंतच्या part चे volume, वृत्तचितीच्या घनफळाच्या formula ने मिळेल. हे घनफळ V मानू.

परंतु $V = \text{volume of the ball (चेंडूचे घनफळ)} + \text{volume of the water which was already in the beaker (आधी भरलेल्या पाण्याचे घनफळ)}$

C) 92430

D) 924300

5) The curved surface area of a frustum of a cone is $21\pi \text{ cm}^2$. If the radii of its upper and lower circular surfaces are 5 cm and 7 cm respectively, then its total surface area is

A) $33\pi \text{ cm}^2$ B) $53\pi \text{ cm}^2$ C) $95\pi \text{ cm}^2$ D) $165\pi \text{ cm}^2$

6) $\frac{\text{Area of sector}}{\text{Area of a circle}} =$

A) $\frac{\theta}{180}$ B) $\frac{\theta}{360}$ C) $\frac{180}{\theta}$ D) $\frac{360}{\theta}$

7) The diameter of a circle is 28 cm. The length of the arc when the corresponding central angle is 45° , is

A) 11 cm

B) 22 cm

C) 44 cm

D) 88 cm

8) If the area of sector and length of the arc of the sector are 54 cm^2 and 18 cm respectively, then the radius of the circle is

A) 3 cm

B) 6 cm

C) 9 cm

D) 36 cm

9) If the measure of an arc of a circle is 36° and its length is 10 cm, then the circumference of the circle is

A) 10 cm

B) 100 cm

C) 36 cm

D) 360 cm

10) If the area of minor segment of a circle of diameter 28 cm is 206 cm^2 then the area of the corresponding major segment is

A) 410 cm^2 B) 412 cm^2 C) 2250 cm^2 D) 2258 cm^2

Ans:

1	B	6	B
2	C	7	A
3	D	8	B
4	C	9	B
5	C	10	A

Additional Problems for Practice

Multiple choice Questions

1) The diameter of a cone whose slant height is 35 cm and curved surface area is 1320 cm^2 is

A) 12 cm

B) 24 cm

C) 36 cm

D) 72 cm

2) If the volume and height of a cone are 3600 cm^3 and 45 cm respectively, then the area of its base is

A) 80 cm^2 B) 160 cm^2 C) 240 cm^2 D) 720 cm^2

3) Volume of two spheres are in the ratio 64 : 27. The ratio of their surface areas is

A) 3:4

B) 4:3

C) 9:16

D) 16:9

4) If capacity of a water reservoir is 92.43 m^3 then it can hold.....litres of water.

A) 92430000

B) 9243

अशाच प्रकारे ...

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